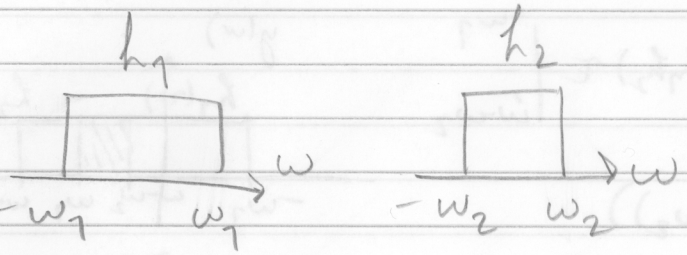
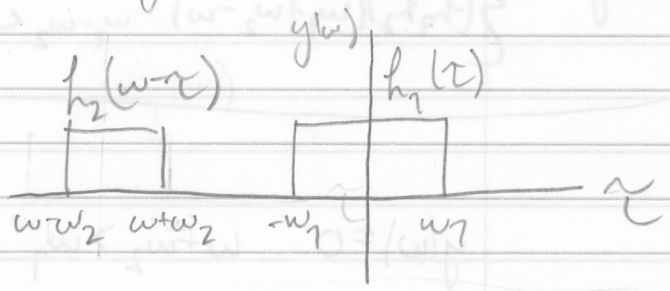
Bonus 6 — Matthew Borja



$\omega_2 \neq \omega_1$

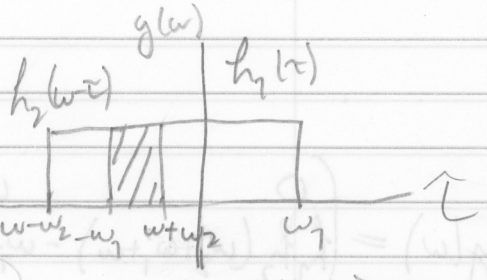
$\omega_2 < \omega_1$

$$h_1 * h_2 = \int_{-\infty}^{\infty} h_1(\tau) h_2(\omega - \tau)\, d\tau = y(\omega)$$



$$y(\omega) = \begin{cases} 0 & \omega+\omega_2 < -\omega_1 \\ & \omega < -\omega_2 - \omega_1 \end{cases}$$

$$y(\omega) = h_1 * h_2 \int_{-\omega_1}^{\omega+\omega_2} (1)\, d\tau = (h_1 * h_2)\,\tau \Big|_{-\omega_1}^{\omega+\omega_2}$$



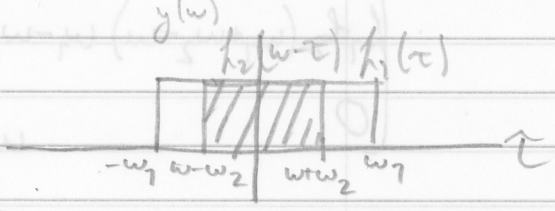
$$= (h_1 * h_2)(\omega+\omega_2) + \omega_1 = (\omega+\omega_2+\omega_1)(h_1 * h_2)$$

$$y(\omega) = \begin{cases} (h_1 * h_2)(\omega+\omega_2+\omega_1) & -\omega-\omega_2 < \omega < \omega_2-\omega_1 \end{cases}$$

$\omega+\omega_2 > -\omega_1$

$\omega-\omega_2 < -\omega_1$

$$y(\omega) = h_1 * h_2 \int_{\omega-\omega_2}^{\omega+\omega_2} (1)\, d\tau = (h_1 * h_2)\,\tau \Big|_{\omega-\omega_2}^{\omega+\omega_2}$$



$$= \left[(\omega+\omega_2) + (\omega+\omega_2)\right] h_1 * h_2 = h_1 * h_2\, 2\omega_2$$

$$y(\omega) = \begin{cases} (h_1 * h_2)\, 2\omega_2 & \omega_2 - \omega_1 < \omega < \omega_1 - \omega_2 \end{cases}$$

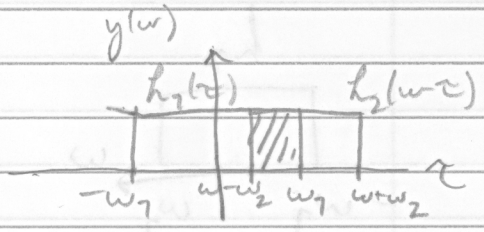
$\omega+\omega_2 < \omega_1$

$\omega-\omega_2 > -\omega_1$

↘

$$y(\omega) = h_1 h_2 \int_{\omega-\omega_2}^{\omega_1} (1)\,d\tau = (h_1 h_2)\tau \Big|_{\omega-\omega_2}^{\omega_1}$$

$$= (h_1 h_2)(\omega_1 - (\omega - \omega_2))$$

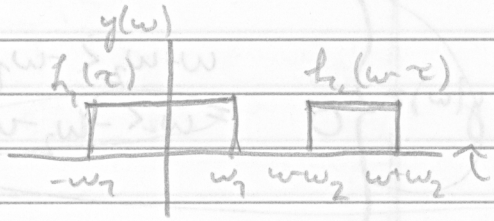


$\omega+\omega_2 > \omega_1$

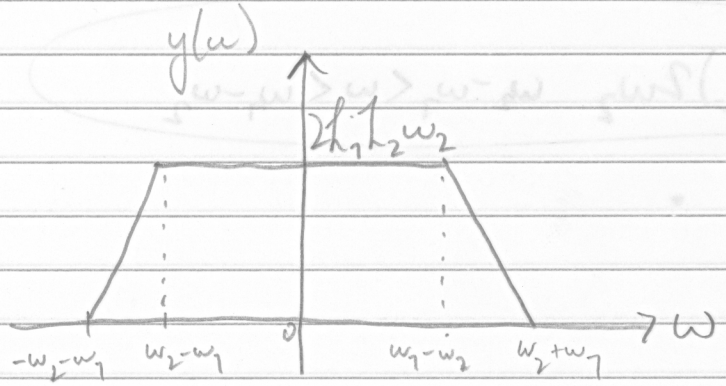
$\omega-\omega_2 < \omega_1$

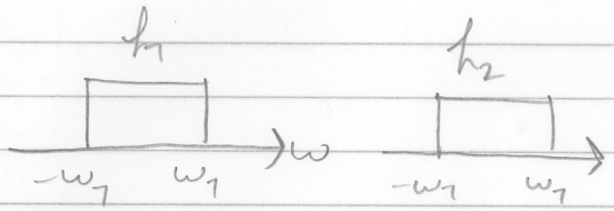
$$y(\omega) = \begin{cases} (h_1 h_2)(\omega_1+\omega_2-\omega) & \omega_1-\omega_2 < \omega < \omega_1+\omega_2 \end{cases}$$

$y(\omega) = 0 \quad \omega-\omega_2 > \omega_1$



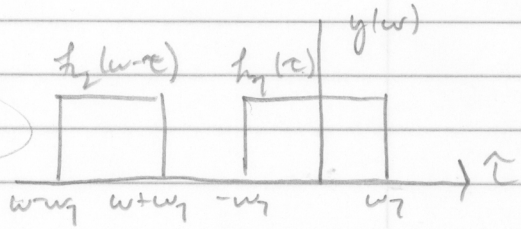
$$y(\omega) = \begin{cases} 0 & \omega < -\omega_1-\omega_2 \\ h_1 h_2(\omega_2+\omega_1+\omega) & -\omega_1-\omega_2 < \omega < \omega_2-\omega_1 \\ h_1 h_2(2\omega_2) & \omega_2-\omega_1 < \omega < \omega_1-\omega_2 \\ h_1 h_2(\omega_1+\omega_2-\omega) & \omega_1-\omega_2 < \omega < \omega_1+\omega_2 \\ 0 & \omega > \omega_1+\omega_2 \end{cases}$$





$$y(\omega) = \int_{-\infty}^{\infty} h_1(\tau) h_2(\omega - \tau)\, d\tau$$

$$y(\omega) = 0 \qquad \omega < -2\omega_1$$

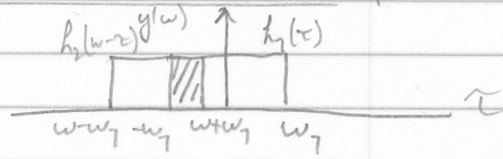


$\omega + \omega_1 < -\omega_1$

$\omega < -2\omega_1$

$$y(\omega) = h_1 h_2 \int_{-\omega_1}^{\omega+\omega_1} (1)\, d\tau = h_1 h_2\, \tau \Big|_{-\omega_1}^{\omega+\omega_1}$$

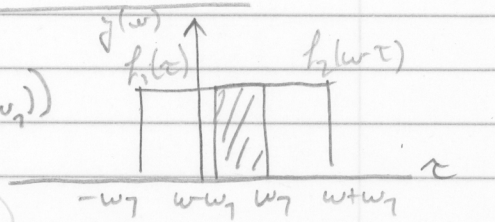
$$= h_1^2 [\omega + \omega_1 + \omega_1]$$



$\omega + \omega_1 > -\omega \qquad \omega - \omega_1 < -\omega$

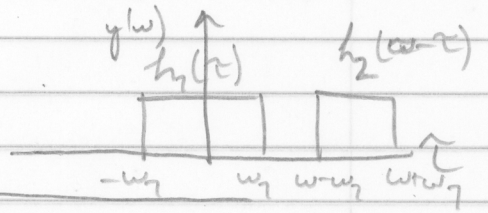
$$y(\omega) = \begin{cases} h_1 h_1 (\omega + 2\omega_1) & -2\omega_1 < \omega < 0 \end{cases}$$

$$y(\omega) = h_1^2 \int_{\omega-\omega_1}^{\omega_1} (1)\, d\tau = h_1^2\, \tau \Big|_{\omega-\omega_1}^{\omega_1} = h_1^2 (\omega_1 - (\omega - \omega_1))$$



$$y(\omega) = \begin{cases} h_1 h_1 (2\omega_1 - \omega) & 0 < \omega < 2\omega_1 \end{cases}$$

$\omega + \omega_1 > \omega_1 \qquad \omega - \omega_1 < \omega_1$



$$y(\omega) = 0 \qquad \omega > 2\omega_1$$

$$y(\omega) = \begin{cases} 0 & \omega < -2\omega_1 \\ h_1^2 (\omega + 2\omega_1) & -2\omega_1 < \omega < 0 \\ h_1^2 (2\omega_1 - \omega) & 0 < \omega < 2\omega_1 \\ 0 & \omega > 2\omega_1 \end{cases}$$