1. Let $f \in BV[0,1], g \in C^1(\mathbb{R})$. Show $g \circ f \in BV[0,1]$.
2. Find the limit of $\int_0^\infty f_n(x)dx$ for
$$f_n(x) = \begin{cases} \dfrac{\sin\left(\dfrac{x+1}{n}\right)}{\sqrt{x}} & \text{if } n \le x \le 2n \\ 0 & \text{otherwise} \end{cases} .$$
3. Prove that there is no Lebesgue measurable subset A of $\mathbb{R}$ such that $a|I| \le |A \cap I| \le b|I|$ for all bounded open intervals $I \subseteq \mathbb{R}$ and $0 < a \le b < 1$. Specifically, prove the following two assertions:
   (a) If $|A \cap I| \le b|I|$ for all open intervals $I \subseteq \mathbb{R}$ and $b < 1$, then $|A| = 0$.
   (b) If $a|I| \le |A \cap I|$ for all open intervals $I \subseteq \mathbb{R}$ and $a > 0$, then $|A| = \infty$.
4. Let $f \in L^1$. Show f^*, the Hardy-Littlewood maximal function is lower semi-continuous. Then prove f^* is measurable. What about $F(x) = \sup \int_Q \frac{|f|}{|Q|}$ where the sup is taken over all cubes containing x?
5. (a) Let $f \in L^1(\mathbb{R})$ and of compact support. Show that for all large $|x|$, the Hardy-Littlewood maximal function, $f^*(x)$ satisfies
$$f^*(x) \le \frac{||f||_1}{|x|}.$$
   (b) Now just suppose $f \in L^1(\mathbb{R})$ and show that for all large $|x|$,
$$f^*(x) \le 1 + \frac{||f||_1}{|x|}$$
.
6. Assume that $f \in AC(I)$ for every $I \subset \mathbb{R}$. If both f and f' are in $L^1(\mathbb{R})$, show that
   (a) $f(x) \to 0$ as $|x| \to \infty$ and
   (b) $\int_{\mathbb{R}} f' = 0$.
7. Let $f : \mathbb{R}_+ \to \mathbb{R}_+$, and let $0 < r < \infty$. Show that
$$\frac{1}{\frac{1}{|I|}\int_I f} \le \left(\frac{1}{|I|}\int_I \frac{1}{f^r}\right)^{1/r}$$
for every $I \subset \mathbb{R}$.

8. Let $C_1(0) = \{f \in L^1[0,1] : ||f||_1 = 1\}$. Show $f \in C_1(0) \Rightarrow f$ is a convex combination of other L^1-unit vectors (i.e. $\exists 0 < \lambda < 1$ and $\exists g_1, g_2 \in C_1(0)$ with $f = \lambda g_1 + (1-\lambda)g_2$. (This result says the unit circle in L^1 is everywhere flat in some sense. Every element of the circle of radius 1 is in the middle of a line connecting 2 other unit vectors!)

9. Let f be continuous on $[-1,1]$. Find

$$\lim_{n\to\infty} n \int_{-1/n}^{1/n} f(x)(1 - n|x|)dx.$$

10. Let X be a metric space and let $A \subset C(X)$ be a linear subspace that contains the constant functions. Suppose also that A is a lattice which separates points. Thus, if f and g are in A, then $f \vee g$ and $f \wedge g$ both belong to A. Let $K \subset X$ be compact and $a \notin K$. Prove that there is an $f \in A$ with $f(a) = 0$ and $f(x) > 1$ for all $x \in K$.