ECE 438 **Assignment No. 6** **Fall 2009**

1. Consider a DT LTI system described by the following equation

$$y[n] = x[n] + 2x[n-1] + x[n-2].$$

Find the response of this system to the input

$$x[n] = \begin{cases} -2, & n = -2, \\ 1, & n = 0, \\ -2 & n = 2, \\ 0, & \text{else.} \end{cases}$$

by the following approaches:

a. directly substitute $x[n]$ into the difference equation describing the system;

b. find the impulse response $h[n]$ and convolve it with $x[n]$;

c. find the frequency response $H(\omega)$ by the following two approaches:

   i. apply the input $e^{j\omega n}$ to the difference equation describing the system,

   ii. find the DTFT of the impulse response,

   verify that both methods lead to the same result, then find the DTFT $X(\omega)$ of the input, multiply it by $H(\omega)$ to yield the DTFT $Y(\omega)$ of the output, and finally calculate the inverse DTFT $y[n]$.

Hints:

i. There is no need to simplify the frequency response or the DTFT of the input.

ii. To evaluate the inverse DTFT of $Y(\omega)$, simply put it in the series form $Y(\omega) = \sum_n y[n]\, e^{-j\omega n}$, and identify the terms $y[n]$ in the series.

d. Verify that all three approaches for finding $y[n]$ lead to the same result.

2. Consider a causal LTI system with transfer function

$$H(z) = \frac{1 - \frac{1}{2} z^{-2}}{1 - \frac{1}{\sqrt{2}} z^{-1} + \frac{1}{4} z^{-2}}$$

a. Sketch the locations of the poles and zeros.

b. Use the graphical approach to determine the magnitude and phase of the frequency response $H(\omega)$, for $\omega = 0, \pi/4, \pi/2, 3\pi/4,$ and π. Based on these values, sketch the magnitude and phase of the frequency response for $-\pi \le \omega \le \pi$. (Be sure to show your work.)

c. Is the system stable, Explain why or why not?

d. Find the difference equation for $y[n]$ in terms of $x[n]$, corresponding to this transfer function $H(z)$.

3. Consider a DT LTI system described by the following *non-recursive* difference equation (moving average filter)

$$y[n] = \frac{1}{8}\{x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4] + x[n-5] + x[n-6] + x[n-7]\}$$

a. Find the impulse response $h[n]$ for this filter. Is it of finite or infinite duration?

b. Find the transfer function $H(z)$ for this filter.

c. Sketch the locations of poles and zeros in the complex z-plane.

Hint: To factor $H(z)$, use the geometric series and the fact that the roots of the polynomial $z^N - p_0 = 0$ are given by

$$z_k = |p_0|^{1/N} e^{j[(\arg p_0)/N + 2\pi k/N]}, k = 0, \ldots, N-1$$

4. Consider a DT LTI system described by the following *recursive* difference equation

$$y[n] = \frac{1}{8}\{x[n] - x[n-8]\} + y[n-1]$$

a. Find the transfer function $H(z)$ for this filter.

b. Sketch the locations of poles and zeros in the complex z-plane.

Hint: See Part c of Problem 3.

c. Find the impulse response $h[n]$ for this filter by computing the inverse ZT of $H(z)$. Is it of finite or infinite duration?

5. Consider the following length 12 sequences:

n	0	1	2	3	4	5	6	7	8	9	10	11
$x_1[n]$	1	1	1	1	1	1	1	1	0	0	0	0
$x_2[n]$	0	1	3	3	4	5	6	7	0	0	0	0

a. Calculate the aperiodic convolution of $x_1[n]$ and $x_2[n]$.

b. Calculate the periodic (period 12) convolution of $x_1[n]$ and $x_2[n]$.

c. To what length would the sequences need to be padded with zeros so that a portion of their periodic convolution would match the nonzero part of their aperiodic convolution?