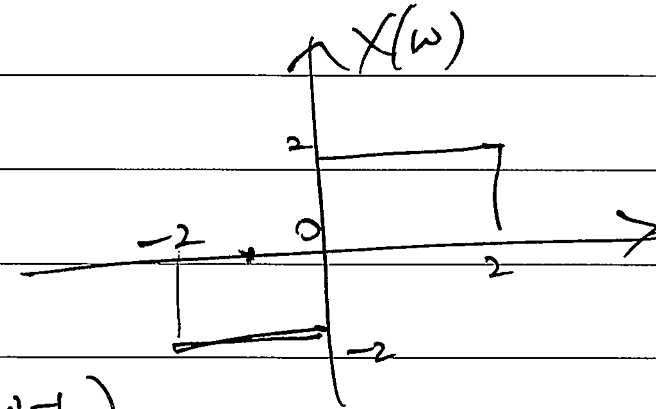
7/14 ①

Example Problems on Fourier Transform using FT pairs and FT properties.

Prob 4.4 (b) (HW4)

Determine I-FT of

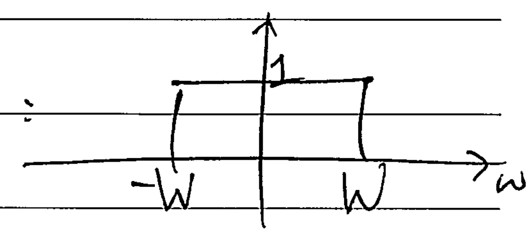


$$X(w) = \begin{cases} 2, & 0 \le w \le 2 \\ -2, & -2 \le w < 0 \\ 0, & |w| > 2 \end{cases}$$

$$= -2\,\mathrm{rect}\left(\frac{w+1}{2}\right) + 2\,\mathrm{rect}\left(\frac{w-1}{2}\right)$$

(Recall) basic FT pair

$$\frac{\sin(Wt)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} \mathrm{rect}\left(\frac{w}{2W}\right) :$$



Consider $W = 1$

* I-FT : Inverse Fourier Transform

Also recall FT property

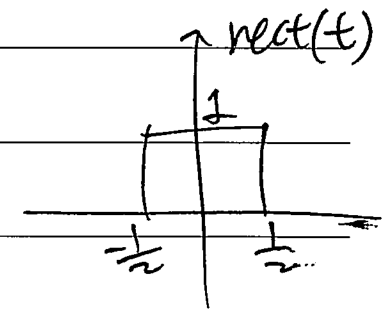
$$e^{j\omega_0 t} x(t) \xleftrightarrow{\mathcal{F}} X(\omega-\omega_0)$$

Consider $\omega_0 = 1$, $\omega_0 = -1$

$$x(t) = \frac{-2\sin(t)}{\pi t} e^{-jt} + \frac{2\sin(t)}{\pi t} e^{jt} \quad \cdots\cdots (1)$$

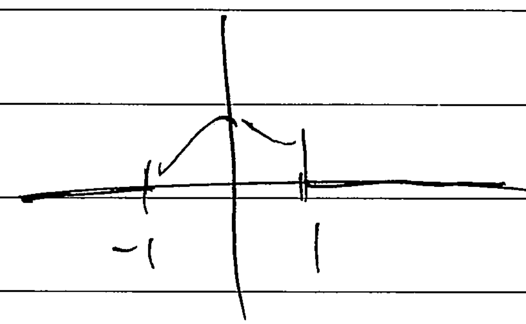
$$= \frac{\sin(t)}{\pi t} \cdot 4j \left\{ \frac{e^{jt} - e^{-jt}}{2j} \right\}$$

$$= 4j \frac{\sin(t)}{\pi t} \sin(t) = 4j \frac{\sin^2(t)}{\pi t}$$



Prob 4.21 (HW4)

(c) $x(t) = \begin{cases} 1+\cos(\pi t) & , |t|<1 \\ 0 & , |t|>1 \end{cases}$



$$= [1+\cos(\pi t)]\, \mathrm{rect}\left(\frac{t}{2}\right)$$

(Recall) Basic FT pair

$$\cdot \ \mathrm{rect}\left(\frac{t}{T}\right) \overset{\mathcal{F}}{\longleftrightarrow} \frac{\sin\left(T\frac{\omega}{2}\right)}{\frac{\omega}{2}}$$

Consider $(T=2)$ &

$$\cdot \ x(t)\cos(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}X(\omega+\omega_0) + \frac{1}{2}X(\omega-\omega_0)$$

$(\omega_0=\pi)$

$(\omega_0=0)$

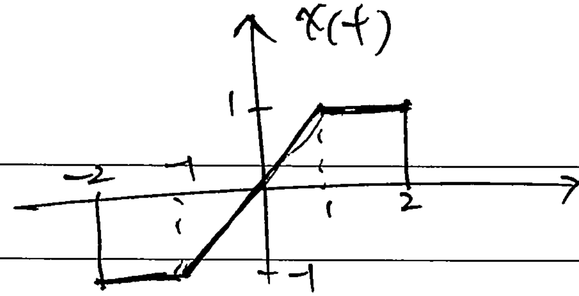
$$X(\omega) = \underbrace{\frac{\sin(\omega)}{\frac{\omega}{2}}}_{\uparrow \mathrm{FT}\ \mathrm{rect}\left(\frac{t}{2}\right)} + \underbrace{\frac{1}{2}\frac{\overbrace{\sin(\omega+\pi)}^{(=-\sin(\omega))}}{\frac{(\omega+\pi)}{2}} + \frac{1}{2}\frac{\overbrace{\sin(\omega-\pi)}^{(=-\sin(\omega))}}{\frac{(\omega-\pi)}{2}}}_{\uparrow \mathrm{FT}\ \mathrm{rect}\left(\frac{t}{2}\right)\cos(\pi t)}$$

$$= \sin(\omega)\left\{\frac{2}{\omega} - \frac{1}{\omega+\pi} - \frac{1}{\omega-\pi}\right\}$$

$$= \sin(\omega)\left\{\frac{2}{\omega} - \frac{2\omega}{\omega^2-\pi^2}\right\}$$

Prob 4.21

(g)



$$x(t) = -\mathrm{rect}\left(\frac{t+\frac{3}{2}}{1}\right) + t\,\mathrm{rect}\left(\frac{t}{2}\right) + \mathrm{rect}\left(\frac{t-\frac{3}{2}}{1}\right)$$

(Recall) FT pair: $\mathrm{rect}\left(\frac{t}{\tau}\right) \overset{\mathcal{F}}{\longleftrightarrow} \dfrac{\sin\left(\tau\frac{\omega}{2}\right)}{\frac{\omega}{2}}$

FT property: $x(t-t_0) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t_0} X(\omega)$

$t\,x(t) \overset{\mathcal{F}}{\longleftrightarrow} j\dfrac{d}{d\omega} X(\omega)$

$$X(\omega) = -\frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}} e^{j\frac{3}{2}\omega} + j\frac{d}{d\omega}\left\{\frac{\sin(\omega)}{\frac{\omega}{2}}\right\} + \frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}} e^{-j\frac{3}{2}\omega}$$

$$= -4j\frac{\sin\left(\frac{\omega}{2}\right)}{\omega}\sin\left(\frac{3}{2}\omega\right) + j2\left\{\frac{\cos(\omega)\,\omega - \sin(\omega)}{\omega^2}\right\}$$

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- other variation on modulation property:

$$x(t)\sin(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2j}X(\omega-\omega_0) - \frac{1}{2j}X(\omega+\omega_0)$$

4.21 (b) $e^{-3|t|}\sin(2t) \xleftrightarrow{\mathcal{F}} X(\omega) = ?$

FT pair: $e^{-a|t|} \xleftrightarrow{\mathcal{F}} \dfrac{2a}{\omega^2+a^2}$

$$X(\omega) = \frac{1}{2j}\frac{2(3)}{(\omega-2)^2+3^2} - \frac{1}{2j}\frac{2(3)}{(\omega+2)^2+3^2}$$

$$= -3j\left\{\frac{1}{(\omega-2)^2+3^2} - \frac{1}{(\omega+2)^2+3^2}\right\}$$

<u>Prob. 4.39</u> Duality Property

If $x(t) \xleftrightarrow{\mathcal{F}} X(\omega)$

then $X(t) \xleftrightarrow{\mathcal{F}} 2\pi \cdot x(-\omega)$

For example: $\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{\mathcal{F}} \dfrac{\sin\left(T\frac{\omega}{2}\right)}{\frac{\omega}{2}}$ dual

Thus: $\dfrac{\sin\left(T\frac{t}{2}\right)}{\frac{t}{2}} \xleftrightarrow{\mathcal{F}} 2\pi \cdot \text{rect}\left(\frac{-\omega}{T}\right)$

We previously showed with $T = 2W$

$$\frac{\sin(Wt)}{\pi t} \longleftrightarrow \text{rect}\left(\frac{\omega}{2W}\right)$$

For part (b), alternative derivation of the FT of sinewave $e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} 2\pi\,\delta(\omega-\omega_0)$

using Duality Property and basic FT pair.

$$\delta(t) \xleftrightarrow{\mathcal{F}} 1$$

$$\delta(t+\beta) \xleftrightarrow{\mathcal{F}} e^{j\beta\omega}\cdot 1 = e^{j\beta\omega}$$

dual

$$e^{j\beta t} \xleftrightarrow{\mathcal{F}} 2\pi\cdot\delta(-\omega+\beta)$$

Note that Dirac delta func. is symmetric $\left(\begin{array}{c}\delta(-t)=\delta(t)\\ \delta(-\omega)=\delta(\omega)\end{array}\right)$

$$e^{j\beta t} \xleftrightarrow{\mathcal{F}} 2\pi\,\delta(\omega-\beta)$$

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