BLA

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Pre-note: I find dealing with all the bounded/unbounded cases boring. So, I'm sticking to the bounded cases. Also, I skipped 3b, because...well, I didn't want to do it.

1) The solution should be the same as number 3 of the previous set.

Check my previous solution.

2) Let $Aj \subseteq R$ and $\alpha_j = \sup_j Aj$, we show that $\sup(\cup_j Aj) = \sup\{\alpha_j\}$.

i) First we verify that $\sup\{\alpha_j\}$ is an upper bound of $\cup_j A_j$

Assume not, then $\exists a \in \cup_j A_j \ni a \geq \alpha_j \forall j$

But if $a \in \cup_j A_j \exists i \ni a \in A_i$ and $a \geq \alpha_i$, but this contradicts that $\alpha_i = \sup(A_i)$.

done.

ii) $\sup\{\alpha_j\}$ is the LEAST upper bound for $\cup_j A_j$

Assume $\cup_j A_j \leq a < \sup\{\alpha_j\}$

Then a is an upper bound $\forall A_j$

So, $\forall j \alpha_j \leq a$

Then a is an upper bound of $\cup\{\alpha_j\}$

$\therefore \sup(\{\alpha_j\}) \leq a$

Contradiction.

$\therefore \sup(\{\alpha_j\})$ is the lub.

done.

Thus by definition $\sup(\cup_j Aj) = \sup\{\alpha_j\}$, which is what we were trying to show.

$\square$

3) Let $a_j \in \mathbf{R}, A_N = \{a_N, a_{N+1}, \cdots\}$

a)

i) We show that $\inf A_N \leq \inf A_{N+1}$

This should follow mutatis mutandis to ii shown below (but 1 and 2 need to be constructed with inf's)

Done.

ii) We show that $\sup A_N \geq \sup A_{N+1}$

Let $A_{N+1} = A_1, \{a_N\} = A_2$

Let $\alpha_1 = \sup(A_1)$.

Apply number 1 to get $\sup(A_2) = a_N$

Note that $A_N = A_{N+1} \cup \{a_N\}$, apply number 2, and thus $\sup A_N = \sup\{\alpha_1, a_N\}$. By 1

again we have the two cases:

$\sup A_N = \alpha_1$ or $\sup A_N = a_N$.

I) $\sup A_N = \alpha_1$ is the trivial case where we satisfy $\sup A_N \geq \sup A_{N+1}$.

Termino.

II) Let $\sup A_N = \sup\{\alpha_1, a_N\} = a_N$

Still pretty trivial but since a_N is the supremum we have $a_n \geq \alpha_1$, which subtituting the correct

expressions in $\sup A_N \geq \sup A_{N+1}$.

Termino.

Thus this shows that $\sup A_N \geq \sup A_{N+1}$.

Done.

□

4)

a) We show that $\oplus_{\mathbf{N}}\mathbf{Q} = \{(q_1, q_1, \cdots) \in \mathbf{Q} \times \mathbf{Q} \times \cdots :$ finitely many q_i are non-zero $\}$

This is countable, because $\mathbf{Q}$ is countable by the corollary to 2.13, and then by theorem 2.13 itself.

□

b) We show that $\Pi_{\mathbf{N}}\mathbf{Q} = \{(q_1, q_1, \cdots) \in \mathbf{Q} \times \mathbf{Q} \times \cdots\}$ is uncountable.

Notice that $A = \{0, 1\} \subseteq \mathbf{Q}$ and thus $\Pi_{\mathbf{N}}A \subseteq \Pi_{\mathbf{N}}\mathbf{Q}$. But then A is the set of all sequences whose elements are digits 0 and 1. Thus, by theorem 2.14 A is uncountable, and since $A \subseteq \mathbf{Q}$, then by theorem

2.8 (it states that no uncountable set can be a subset of a countable end by the end of the proof) $\Pi_{\mathbf{N}}\mathbf{Q}$ is uncountable.

□