Time and Frequency Shifting Property of the Fourier Transform

- From Table 4.1, it is stated that the Fourier Transform of a time shifted signal, $x(t-t_0)$ is equals to

$$F\{x(t-t_0)\} = e^{-j\omega t_o}X(\omega)$$

Let $x(t) = \delta(\omega)$,

$$F\{x(t)\} = F\{\delta(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt = 1$$

Let $x(t-t_0) = \delta(t-t_0)$,

$$F\{x(t-t_0)\} = F\{\delta(t-t_0)\} = \int_{-\infty}^{\infty} x(t-t_0)e^{-j\omega t}dt = \int_{-\infty}^{\infty} \delta(t-t_0)e^{-j\omega t}dt = e^{-j\omega t_0}\cdot 1$$

$$\therefore F\{x(t-t_0)\} = e^{-j\omega t_o}F\{x(t)\} = e^{-j\omega t_o}X(\omega)$$

- Also, from Table 4.1, if a Fourier Transform signal were to be shifted by ω_0, the Inverse Fourier Transform would be,

$$F^{-1}\{X(\omega-\omega_0)\} = e^{-j\omega_0 t}x(t)$$

Let $X(\omega) = \delta(\omega)$,

$$F\{X(\omega)\} = F\{\delta(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{-j\omega t}dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)e^{-j\omega t}dt = \frac{1}{2\pi}$$

Let $X(\omega-\omega_0) = \delta(\omega-\omega_0)$,

$$F\{X(\omega-\omega_0)\} = F\{\delta(\omega-\omega_0)\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega-\omega_0)e^{-j\omega t}dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)e^{-j\omega t}dt = e^{-j\omega_0 t}\cdot\frac{1}{2\pi}$$

$$\therefore F^{-1}\{X(\omega-\omega_0)\} = e^{-j\omega_0 t}F\{X(\omega)\} = e^{-j\omega_0 t}x(t)$$