

Student Name: _____

S/n.

Grader Name: _____

Instructions: This is an open notes, open book, collaborative quiz. No Internet allowed.

Question: Let X be a one-dimensional normal random variable with mean equal to three and standard deviation equal to two. Use the standard normal table attached to compute the following probabilities

- a) Prob ($X < 3.08$)
- b) Prob ($X > 3.02$)
- c) Prob ($3.02 < X < 3.08$)
- d) Prob ($0.98 < X < 1.18$)

(Give the numerical values for each probability and write the intermediate steps you followed to get these values.)

Answer:

$Y = \frac{X-3}{2}$ is a standard normal variable

$$a) \text{Prob}(X < 3.08) = \text{Prob}\left(Y < \frac{3.08-3}{2}\right) = \text{Prob}(Y < 0.04) = \Phi(0.04) \approx 0.5160$$

$$b) \text{Prob}(X > 3.02) = \text{Prob}\left(Y > \frac{3.02-3}{2}\right) = \text{Prob}(Y > 0.01) = 1 - \Phi(0.01) \\ \approx 1 - 0.5040 \\ = 0.4960$$

$$c) \text{Prob}(3.02 < X < 3.08) = \text{Prob}(0.01 < Y < 0.04) \\ = \Phi(0.04) - \Phi(0.01) \\ \approx 0.5160 - 0.5040 = 0.0120$$

$$d) \text{Prob}(0.98 < X < 1.18) = \text{Prob}\left(\frac{0.98-3}{2} < Y < \frac{1.18-3}{2}\right) \\ = \text{Prob}\left(-\frac{2.02}{2} < Y < -\frac{2.18}{2}\right) \\ = \text{Prob}(-1.01 < Y < -1.09) \\ = \Phi(-1.09) - \Phi(-1.01) \\ = \Phi(-1.09) - (1 - \Phi(1.09))$$

g the fact

$$2 dy = 1,$$

the normal PDF for the case where
 special properties. The following
 stified in Section 4.1.

transformations

an μ and variance σ^2 , and if $a \neq 0$,

-b

$$f_Y(Y) = a^2 \sigma^2.$$

ble

an and unit variance is said to be a
 Φ :

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^y e^{-t^2/2} dt.$$

it page), and is a very useful tool
 normal random variables; see also

values of $\Phi(y)$ for $y \geq 0$, because the
 entry of the PDF. For example, if Y
 ive

$$\Phi(-0.5) = 1 - \Phi(0.5) = 0.3085.$$

for all y .

	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9997	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

The standard normal table. The entries in this table provide the numerical values of $\Phi(y) = P(Y \leq y)$, where Y is a standard normal random variable, for y between 0 and 3.49. For example, to find $\Phi(1.71)$, we look at the row corresponding to 1.7 and the column corresponding to 0.01, so that $\Phi(1.71) = .9564$. When y is negative, the value of $\Phi(y)$ can be found using the formula $\Phi(y) = 1 - \Phi(-y)$.