Thursday, September 06, 2007
3:20 PM

$y[n] = x[n] * h[n]$ for $\boxed{\boxed{LTI}}$

Properties of LTI systems

Property 1 $\quad x(t) \to \boxed{h(t)} \to y(t)$

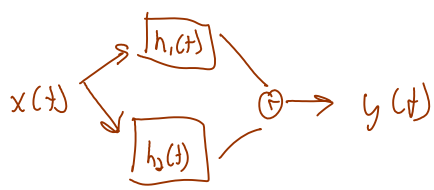
$h(t) \to \boxed{x(t)} \to y(t)$

then system yield same output

why! because convolution is communitatin

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

Property 2



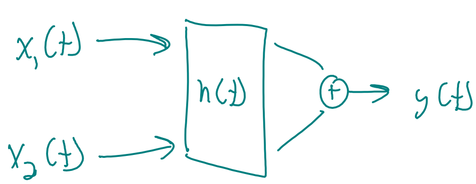
yields same as

$x(t) \to \boxed{h_1(t) + h_2(t)} \to y(t)$

because "*" is distributative

$$x_1(t) * (x_2(t) + x_3(t)) = x_1(t)x_2(t) + x_1(t)x_3(t)$$

Property 3



yields same as

$x_1(t) + x_2(t) \to \boxed{h(t)} \to y(t)$

because by distributivity an commutability of "*"

$$(x_1(t) + x_2(t)) * h(t) = h(t) * (x_1(t) + x_2(t)) = h(t)x_1(t) + h(t)x_2(t)$$

Property 4

$x[n] \to \boxed{h_1[n]} \to \boxed{h_2[n]} \to y[n]$ yields same

$x[n] \to \boxed{h_1[n] * h_2[n]} \to y[n]$ also

$x[n] \to \boxed{h_2[n]} \to \boxed{h_1[n]} \to y[n]$

LTI Systems with or without memory

<u>Fact 1</u> If an LTI system is memoryless than its unit impulse response can be written as:

DT: $h[n] = cst\ \delta[n]$ $cst \in \mathbb{C}$ (complex)

CT: $h(t) = cst\ \delta(t)$

why? because $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

so if $y[n]$ does not depend on $x[k]$ for $k>n$ or $k>n$

so $h[n-k]$ must be zero when $k>n$ or $k<n$

If system is LTI and $h[n]$ is not zero for all $n \neq 0$ ($h(t)$ is not zero for all $t \neq 0$) then system has memory

Corollary: If system is LTI and memoryless, $y[n] = cst\ x[n]$

Fact 2 If an LTI signal is invertable, than its inverse is LTI

Corollary: The unit impulse response $h(t)$ ($h[n]$) of a system and the unit impulse response $\bar{h}(t)$ ($\bar{h}[n]$) of its inverse, satisfy

$$h(t) * \bar{h}(t) = \delta(t) \quad \text{or} \quad h[n] * \bar{h}[n] = \delta[n]$$