Midterm Examination 1
ECE 301 - Spring 2019
Instructor: Prof. Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.

2. You have 50 minutes to complete this exam. **When the end of the exam is announced, you must stop writing and close this booklet immediately**. Failure to do so will result in a grade of zero on this exam.

3. At the end of this document is a table of formulas and some scratch paper. You may detach these **once the exam begins** but you must slide them all back inside your exam before handing it in. Failure to do so will result in a 5 point penalty on the exam.

4. Except for brief looks to check the time remaining, you are expected to keep your eyes on your exam at all times. No looking around anywhere except on your exam. Thank you for your collaboration.

5. This is a closed book exam. No calculators. All electronic communication devices (Cell phones, watches, etc.) must be turned off and stowed away in your bag (not in your pocket) at all times.

Name: [redacted]

Email: [redacted]

Signature: [redacted]

Itemized Scores

Problem 1: 9	Problem 4: 5
Problem 2: 19	Problem 5: 22
Problem 3: 12	Problem 6: 20
Total:	

9

(20 pts) **1.** Compute the energy E_∞ and the power P_∞ of the discrete-time signal

$$x[n] = \begin{cases} j\left(\frac{1}{3}\right)^n, & \text{if } n \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

(Show the steps of your computations.)

$$E_\infty = \sum_{n=1}^{\infty} |x[n]|^2 = \sum_{n=1}^{\infty} \left|j\left(\tfrac{1}{3}\right)^n\right|^2$$

norm

$$= \sum_{n=1}^{\infty} \left(\left(\tfrac{1}{3}\right)^n\right)^2$$

9

$$= \sum_{n=1}^{\infty} \left(\tfrac{1}{9}\right)^n \text{ geoseries}$$

$$\tfrac{1}{8} \quad = \frac{1}{1-\frac{1}{9}} - \tfrac{1}{9} = E_\infty$$

$$-1$$

$$E_\infty = \cancel{\tfrac{9}{8}} + \cancel{\tfrac{1}{9}} = \cancel{\tfrac{74}{9}} = E_\infty$$

$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\tfrac{1}{9}\right)^n \quad \cancel{\left(\tfrac{1}{9}\right)^N = 0 \quad \left(\tfrac{1}{9}\right)^{-N} = \infty}$$

use finite geo. sum

$$\cancel{\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\tfrac{1}{9}\right)^n = \frac{1}{2(\infty)+1} \cdot \infty = 0}$$

(20 pts) **2.** Is the system given by the equation

$$y(t) = x(2t)$$

time-invariant? Justify ok. (i.e., prove) your answer.

$$x(t) \rightarrow \boxed{\text{SYS}} \rightarrow \boxed{\underset{\text{by } t_0}{\text{TD}}} \Rightarrow x(2t - 2t_0)$$

$$x(t) \rightarrow \boxed{\underset{\text{by } t_0}{\text{TD}}} \rightarrow \boxed{\text{SYS}} \Rightarrow x(2t - t_0)$$

not TI

19

clarity/sign −1

(15 pts) **3.** True or False? Circle the correct answer. (no justification needed)

a) Every memoryless system is causal. **True/False**

b) Every linear system is time-invariant **True/False**

12 c) The unit impulse response of an LTI system is a Dirac delta. **True/False**

d) The signal $x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{1+(t+\frac{k}{3})^4}$ is periodic. **True/False**

e) A stable system is a system whose output is always bounded. **True/False**

(10 pts) **4.** A discrete-time LTI system has unit impulse response

$$\delta[n-5]$$

What is the system's response to the input $x[n] = 3^n$? (Briefly justify your answer.)

$h[n] * x[n] = 3^N \delta[n-5-N]$

convolution ≠ multiplication Write the proper formula!

the only time $\delta \neq 0$ @ $N = n-5$ ∴

$= 3^{n-5}$

5

(30 pts) **5.** A discrete-time LTI system has unit impulse response

$$h[n] = u[-n].$$

Compute the system's response to the input $x[n] = 2^n u[-n]$. (Show the steps of your computation.)

$$h[n] * x[n] = \sum_{K=-\infty}^{\infty} x[K]\, h[n-K]$$

change bounds

$$= \sum_{K=-\infty}^{\infty} 2^K u[-K]\, u[-N+K]$$

$$= \sum_{K=-\infty}^{0} 2^K u[-N+K]$$

placement makes it difficult to read.

if $K > N$

$$= \sum_{K=N}^{0} 2^K$$

if $\underline{N} > K$

$= 0$

$$= \sum_{K=N}^{0} \frac{1}{2}^{-K}$$

$$h[n] * x[n] = \frac{1 - \frac{1}{2}^{n+1}}{1 - \frac{1}{2}}$$

$N \neq n.$

22

(30 pts) **6.** A continuous-time LTI system has unit impulse response

$$h(t) = e^{4t}u(t).$$

Compute the system's response to the input $x(t) = u(t) - u(t-5)$. (Show the steps of your computation.)

$$h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$u(\tau) - u(\tau - 5)$

change boundaries

$$= \int_0^5 h(t-\tau)\, d\tau$$

if $t < \tau$

$h(t) * x(t) = 0$

$$= \int_0^5 e^{4(t-\tau)}\, u(t-\tau)\, d\tau \qquad t \geq \tau$$

$$= \int_0^5 e^{4(t-\tau)}\, d\tau$$

$$= e^{4t} \int_0^5 e^{-4\tau}$$

$$= e^{4t} \left| \frac{e^{-4\tau}}{-4} \right|_0^5$$

$$\left(\frac{e^{-20}}{-4} - \frac{1}{4} \right) e^{4t}$$

$$h(t) * x(t) = \left(\frac{e^{4t-20}}{-4} - \frac{e^{4t}}{4} \right)$$

10

there should be 3 cases.

Facts and Formulas

1 CT Signal Energy and Power

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \tag{1}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \tag{2}$$

2 DT Signal Energy and Power

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 \tag{3}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \tag{4}$$

Midterm Examination 2
ECE 301
Spring 2019
Instructor: Prof. Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. You have 50 minutes to complete the 7 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
3. The last pages contain a table of formulas and properties and some scratch paper. You may tear out these pages **once the exam begins**. However, every single page you tore out must be handed back, tucked inside your exam. Failure to do so will result in a 5 point penalty on the exam.
4. Electronic communications devices are strictly prohibited in the classroom during the exam. They must be turned off and stowed away in your bag (not in your pocket!). This includes wifi/bluetooth enabled watches.

Name: [redacted]

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Itemized Scores

Problem 1+2: 24
Problem 3: 6
Problem 4: 5
Problem 5: 20
Problem 6: 0
Problem 7: 20
Total: 75

(15 pts) **1.** An LTI system is defined by the equation

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

a) What is the unit impulse response of that system? (briefly justify your answer) replace $x(\tau)$ w/ $\delta(\tau)$

$$\Rightarrow h(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$
$$= u(t)$$

$\cancel{\text{if } t \neq 0 \therefore \int_{-\infty}^{t} \delta(\tau)d\tau = u(\tau)}$

$\cancel{h(\tau)} = u(\tau)$

why τ? dimming the variable is t

b) Use your response in a) to determine if the system is stable.

$$\int_{-\infty}^{\infty} |h(t)|\, dt$$
$$= \int_{-\infty}^{\infty} u(t)\, dt$$
$$= \infty \therefore \text{not stable}$$

15

(10 pts) **2.** Is the statement below true or false? (Briefly justify your answer)
Let $x(t)$ be a real signal. Let $\mathcal{X}(\omega) = F_1(\omega) + jF_2(\omega)$ be the CTFT of $x(t)$. Then the imaginary part $F_2(\omega)$ of $\mathcal{X}(\omega)$ satisfies $F_2(0) = 0$.

True

$x(t)$ real $\Rightarrow$ Im(odd)

~~for odd equations~~ fcts

~~$x(-0) = -x(0)$~~

~~b/c~~

$\Rightarrow F_2(0) = 0$

$\frac{9}{10}$ (-1 messy)

(15 pts) **3.** Obtain the Fourier series coefficients of the CT signal

$$x(t) = \cos(2\pi 440 t).$$

Justify your answer.

$$\cancel{\mathcal{F}(\cos(2\pi 440 t)) = \mathcal{F}\left(\frac{e^{j2\pi 440 t}}{2} + \frac{e^{-j2\pi 440 t}}{2}\right)}$$

$$\cancel{= \int_{-\infty}^{\infty} \left(\frac{e^{j2\pi 440 t}}{2} + \frac{e^{-j2\pi 440 t}}{2}\right) e^{-j\omega t}\, dt}$$

$a_1 = a_{-1} = \frac{1}{2}$ $\quad a_k = 0$ repeats every

$a_{k+1/440} = a_k$

6

(20 pts) **4.** Obtain the Fourier series coefficients of the DT signal

$$x[n] = \sin\left(\frac{\pi n}{5}\right).$$

Justify your answer.

$$x[n] = \frac{1}{2j} e^{j\frac{\pi}{5}n} - \frac{1}{2j} e^{-j\frac{\pi}{5}n}$$

3

$$x[n]\ \cancel{a_K} = \frac{1}{2j} e^{j\frac{2\pi}{10}n} - \frac{1}{2j} e^{-j\frac{2\pi}{10}n} \cdot \left(e^{j2\pi n}\right)$$

$$\Rightarrow a_K = \frac{1}{2j} e^{j\frac{2\pi}{10}n} - \frac{1}{2j} e^{j\frac{8\pi}{10}n} \quad 9 \cdot \frac{2\pi}{10}n$$

$$a_1 = \frac{1}{2j}, \quad a_8 = -\frac{1}{2j} \qquad a_K = 0$$

$$a_{K+10} = a_K$$

5

(20 pts) **5.** What is the CTFT of the signal

$$x(t) = e^{3t}u(-t+1)?$$

Justify your answer.

$$X(\omega) = \mathcal{F}(x(t)) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{3t} u(-t+1) e^{-j\omega t} dt$$

$u(-t+1) = 1$

change bounds

$$= \int_{-\infty}^{1} e^{(3-j\omega)t} dt$$

$$= \left. \frac{e^{(3-j\omega)t}}{(3-j\omega)} \right|_{-\infty}^{1}$$

$$= \frac{e^{(3-j\omega)1}}{(3-j\omega)} - 0$$

$$\mathcal{F}(x(t)) = \frac{e^{(3-j\omega)}}{3-j\omega}$$

20

(30 pts) **6.** What is the CTFT of the signal

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 10k)?$$

Justify your answer.

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-10k)$$

$$\mathcal{F}(x(t)) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(t-10k)\, e^{-j\omega t}\, dt$$

$\delta \neq 0$ @ $t = 10k$

$\therefore$

$$\mathcal{F}(x(t)) = \sum_{k=-\infty}^{\infty} e^{-j\omega 10k}$$

diverges.

0/30

(20 pts) **7.** Consider a discrete-time LTI system with unit impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$. Use Fourier transforms to obtain the system's response to the input signal $x[n] = \left(\frac{1}{4}\right)^n u[n]$. Show the steps of your computation.

$$\mathcal{F}(h[n]) = H(\omega) \qquad \mathcal{F}(x[n]) = X(\omega)$$

$$H(\omega) \cdot X(\omega) = Y(\omega)$$

$$\mathcal{F}(h[n]) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2e^{j\omega}}$$

geo

$$\mathcal{F}(h[n]) = \frac{1}{1 - \frac{1}{2e^{j\omega}}} = H(\omega)$$

use table!

$$\mathcal{F}(x[n]) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4e^{j\omega}}\right)^n$$

geo

$$\mathcal{F}(x[n]) = \frac{1}{1 - \frac{1}{4e^{j\omega}}} = X(\omega)$$

use table!

$$Y(\omega) = \frac{1}{\left(1 - \frac{1}{2e^{j\omega}}\right)\left(1 - \frac{1}{4e^{j\omega}}\right)}$$

$$\frac{1}{\left(1 - \frac{1}{2e^{j\omega}}\right)\left(1 - \frac{1}{4e^{j\omega}}\right)} = \frac{A}{\left(1 - \frac{1}{2e^{j\omega}}\right)} + \frac{B}{\left(1 - \frac{1}{4e^{j\omega}}\right)}$$

$$1 = \left(1 - \frac{1}{4e^{j\omega}}\right)A + \left(1 - \frac{1}{2e^{j\omega}}\right)B$$

$$A = 2 \qquad B = -1$$

$$\therefore \quad Y(\omega) = \frac{2}{\left(1 - \frac{1}{2e^{j\omega}}\right)} + \frac{-1}{\left(1 - \frac{1}{4e^{j\omega}}\right)}$$

inverse DTFT

$$y[n] = 2\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n]$$

20

Table

1 Definition of the Continuous-time Fourier Transform

Let $x(t)$ be a signal and denote by $\mathcal{X}(\omega)$ its Fourier transform.

$$\text{Fourier Transform: } \mathcal{X}(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \tag{1}$$

$$\text{Inverse Fourier Transform: } x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{X}(\omega)e^{j\omega t}d\omega \tag{2}$$

2 Properties of the Continuous-time Fourier Transform

Let $x(t)$ be a continuous-time signal and denote by $\mathcal{X}(\omega)$ its Fourier transform. Let $y(t)$ be another continuous-time signal and denote by $\mathcal{Y}(\omega)$ its Fourier transform.

$$\text{Linearity: } ax(t) + by(t) \xrightarrow{\mathcal{F}} a\mathcal{X}(\omega) + b\mathcal{Y}(\omega) \tag{3}$$

$$\text{Time Shifting: } x(t - t_0) \xrightarrow{\mathcal{F}} e^{-j\omega t_0}\mathcal{X}(\omega) \tag{4}$$

$$\text{Frequency Shifting: } e^{j\omega_0 t}x(t) \xrightarrow{\mathcal{F}} \mathcal{X}(\omega - \omega_0) \tag{5}$$

$$\text{Conjugation: } x^*(t) \xrightarrow{\mathcal{F}} \mathcal{X}^*(-\omega) \tag{6}$$

$$\text{Scaling: } x(at) \xrightarrow{\mathcal{F}} \frac{1}{|a|}\mathcal{X}\left(\frac{\omega}{a}\right) \tag{7}$$

$$\text{Multiplication: } x(t)y(t) \xrightarrow{\mathcal{F}} \frac{1}{2\pi}\mathcal{X}(\omega) * \mathcal{Y}(\omega) \tag{8}$$

$$\text{Convolution: } x(t) * y(t) \xrightarrow{\mathcal{F}} \mathcal{X}(\omega)\mathcal{Y}(\omega) \tag{9}$$

$$\text{Differentiation in Time: } \frac{d}{dt}x(t) \xrightarrow{\mathcal{F}} j\omega\mathcal{X}(\omega) \tag{10}$$

$$x(t)\text{real} \xrightarrow{\mathcal{F}} \mathcal{X}(\omega) = \mathcal{X}^*(-\omega) \tag{11}$$

$$x(t)\text{real and even} \xrightarrow{\mathcal{F}} \mathcal{X}(\omega)\text{real and even} \tag{12}$$

$$x(t)\text{real and odd} \xrightarrow{\mathcal{F}} \mathcal{X}(\omega)\text{pure imaginary and odd} \tag{13}$$

$$\text{Parseval's Relation: } \int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\mathcal{X}(\omega)|^2\, d\omega \tag{14}$$

3 Fourier Series of Continuous-time Periodic Signals with period T

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t} \tag{15}$$

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\left(\frac{2\pi}{T}\right)t}dt \tag{16}$$

4 Fourier Series of Discrete-time Periodic Signals with period N

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\left(\frac{2\pi}{N}\right)n} \tag{17}$$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\left(\frac{2\pi}{N}\right)n} \tag{18}$$

5 Definition of the Discrete-time Fourier Transform

Let $x[n]$ be a discrete-time signal and denote by $\mathcal{X}(\omega)$ its Fourier transform.

$$\text{Fourier Transform: } \mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{19}$$

$$\text{Inverse Fourier Transform: } x[n] = \frac{1}{2\pi}\int_{2\pi} \mathcal{X}(\omega)e^{j\omega n}d\omega \tag{20}$$

6 Some Discrete-time Fourier Transforms

$$\delta[n] \xrightarrow{\mathcal{F}} 1 \tag{21}$$

$$\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{1-\alpha e^{-j\omega}} \tag{22}$$

$$(n+1)\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{(1-\alpha e^{-j\omega})^2} \tag{23}$$