Practice for Midterm Examination 2
ECE 302, Spring 2013
Instructor: Prof. Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.

2. You have 50 minutes to complete this exam. You must remain sitting until the end of the exam is announced. **You may not leave the exam early.** When the end of the exam is announced, you **must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.

3. Do **not** tear out any page from this booklet.

4. You must keep your eyes on your desk at all times. Looking around is not allowed.

5. This is a closed book exam. The use of calculators is prohibited. Cell phones, iPods, and all other electronic communication devices are strictly forbidden. This means that they MUST BE TURNED OFF (not on vibrate mode) and stowed away (in your bag, not in your pocket) AT ALL TIMES.

Name:____

Email:____

Signature:______________________________________

Itemized Scores
Problem 1:
Problem 2:
Problem 3:
Problem 4:
Problem 5:
Total:

(20 pts) **1.** Let X be a discrete random variable with probability mass function

$$p_X(n) = C\frac{1}{3^{|n-1|}}, \text{ for } n \in \mathbf{Z}$$

where C is a constant. Compute the expectation of X. (You must write the intermediate steps of your computation to get full credit.) Hint: Recall the geometric series $\frac{1}{1-r} = \sum_{k=0}^{\infty} r^k$ for any r with $|r| < 1$.

(20 pts) **2.** Let (X, Y) be a 2D random variable that is uniformly distributed inside the curve implicitly defined by the equation

$$\frac{x^2}{4} + y^2 = 1$$

Find the conditional probability density function $f_{X|Y}(x|y)$. (You must clearly explain how you obtained your answer in order to get full credit.)

(20 pts) **3.** Suppose a fair coin is tossed n times. Each coin toss costs d dollars and the reward in obtaining X heads is c^X, where $c > 0$. Find the expected value of the net reward. (You must clearly justify your answer to get full credit.) Hint: Recall the binomial series $(a+b)^n = \sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} a^k b^{n-k}$.

(15 pts) **4.** Two continuous random variables X and Y have the following joint probability density function:

$$f_{XY}(x, y) = Ce^{-\frac{1}{2}(\frac{x^2}{9}+\frac{x^2}{25})},$$

where C is a constant. Are X and Y independent? Answer yes/no and give a mathematical proof of your answer.

(20 pts) **5.** A random variable X has the following probability density function:

$$f_X(x) = \begin{cases} k + \frac{1}{10}x, & \text{if } 0 \le x \le 2, \\ 0, & \text{else,} \end{cases}$$

where k is a constant. Compute $P(1 \le X \le 2)$.

-SCRATCH -
(will not be graded)

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