Homework 4

3.25) $T = 1/2$

(a) $x(t) = \cos(4\pi t)$

$$= \frac{1}{2}\left(e^{j4\pi t} + e^{-j4\pi t}\right)$$

$$= \frac{1}{2} e^{j(1)\left(\frac{2\pi}{1/2}\right)t} + \frac{1}{2} e^{j(-1)\left(\frac{2\pi}{1/2}\right)t}$$

$\therefore\ a_1 = a_{-1} = 1/2$ All other coeff. $= 0$.

(b) $y(t) = \sin(4\pi t)$

$$= \frac{1}{2j}\left(e^{j4\pi t} - e^{-j4\pi t}\right)$$

$$= \frac{1}{2j} e^{j(1)\left(\frac{2\pi}{1/2}\right)t} - \frac{1}{2j} e^{j(-1)\left(\frac{2\pi}{1/2}\right)t}$$

$\therefore\ a_1 = \frac{1}{2j}$

$a_{-1} = \frac{-1}{2j}$

All other co-eff. $= 0$

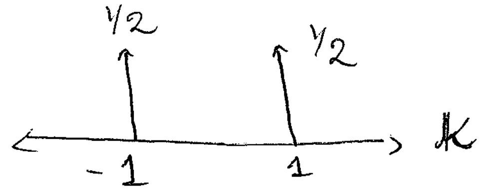
Ans.

(c) $x(t) \xrightarrow{F.S.} a_k$

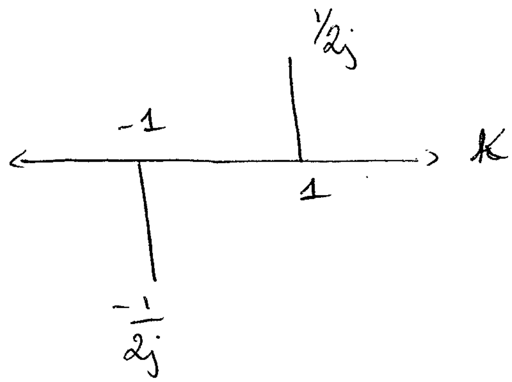
$y(t) \xrightarrow{F.S.} b_k$

$$x(t)\,y(t) \xrightarrow{F.S.} c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

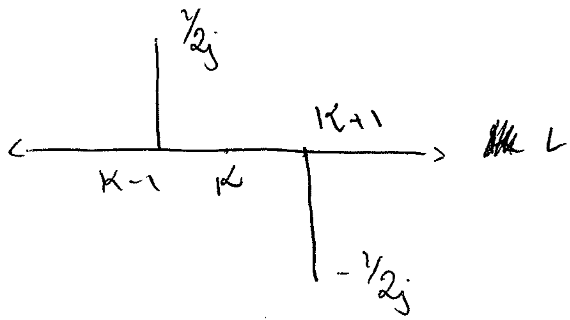
$a_K \rightarrow$



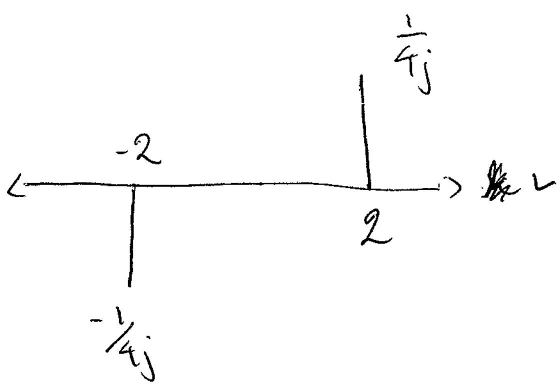
$b_K \rightarrow$



$b_{K-L} \rightarrow$



$C_K \rightarrow$



(d) $z(t) = y(t)\,x(t)$

$= \sin(4\pi t)\cos(4\pi t) \rightarrow 2\sin\theta\cos\theta = \sin 2\theta$

$\sin\theta\cos\theta = \dfrac{\sin 2\theta}{2}$

$= \dfrac{1}{2}\sin(8\pi t)$

$= \dfrac{1}{4j}\left(e^{+j8\pi t} - e^{-j8\pi t}\right)$

$= \dfrac{1}{4j}e^{j(2)(4\pi)t} - \dfrac{1}{4j}e^{j(-2)(4\pi)t}$

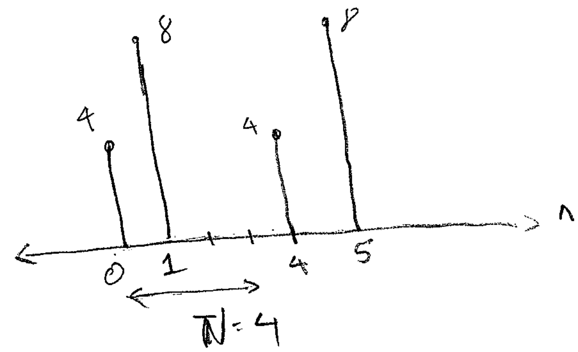
$\therefore \quad a_2 = 1/4j \; ; \quad a_{-2} = \frac{-1}{4j} \; ;$ All other co-eff $= 0$.

Ans:

3.9)

$$x[n] = \sum_{m=-\infty}^{\infty} \left\{ 4\delta[n-4m] + 8\delta[n-1-4m] \right\}$$

$\therefore \quad N = 4 \quad \longleftrightarrow$



$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n}$$

$$= \frac{1}{4} \sum_{n=0}^{3} x[n] e^{-jk 2\pi/4 \, n}$$

$$= \frac{1}{4} \sum_{n=0}^{3} x[n] e^{-jk \pi/2 \, n}$$

$$= \frac{1}{4} \left[4 + 8 e^{-jk\pi/2} \right]$$

$$= 1 + 2e^{-jk\pi/2}$$

$\therefore \quad a_0 = 1 + 2 = 3$

$a_1 = 1 + 2e^{-j\pi/2} = 1 - 2j$

$a_2 = 1 + 2e^{-j\pi} = 1 + 2(-1) = -1$

$a_3 = 1 + 2e^{-j3\pi/2} = 1 + 2j$

3.10) $N = 7$

$x[n] = -x[-n]$ { Odd signal }

$$\left.\begin{aligned} a_{15} &= j \\ a_{16} &= 2j \\ a_{17} &= 3j \end{aligned}\right\} \text{(Given)}$$

$a_0 = 0$ { signal is real & odd. a_K will be imaginary and odd }

$a_1 = a_{15} = j$

But, $a_1 = \cancel{-a_{-1}} \; -a_{-1} = j$

$$\left.\begin{aligned} \therefore \quad a_{-1} &= -j \\ a_{-2} &= -2j \\ a_{-3} &= -3j \end{aligned}\right\} \underline{\text{Ans:}}$$

3.11) 1.) $x[n]$ is a real signal

$x[n] = x[-n]$ { Even signal }

i.e. $a_K = a_{-K}$

2.) $N = 10$

$a_1 = a_{11} = a_{21} = \_\_\_\_$

3.) $a_{11} = 5$

4.) $$\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = 50$$

Prove - $x[n] = A\cos(Bn+C)$

$$\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = \sum_{k=\langle N\rangle}|a_k|^2 = 50$$

$$\sum_{k=\langle -1\rangle}^{8}|a_k|^2 = 50$$

$$|a_{-1}|^2 + |a_0|^2 + |a_1|^2 + \sum_{k=2}^{8}|a_k|^2 = 50$$

$$|a_0|^2 + \cancel{|5|^2} + \cancel{\;} + \cancel{|5|^2} + \sum_{k=2}^{8}|a_k|^2 = \cancel{50}$$

$$|a_0|^2 + \sum_{n=2}^{8}|a_k|^2 = 0$$

$\therefore\ a_k = 0$ for $k = 2, 3, ---, 8.$

$$\therefore\ x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\frac{2\pi}{N}n}$$

$$= \sum_{k=-1}^{8} a_k e^{jk\frac{2\pi}{10}n}$$

$$= 5e^{j\frac{2\pi}{10}n} + \cancel{8}\, 5e^{-j\frac{2\pi}{10}n}$$

$$= 5\times 2\left(\frac{e^{j\pi/5\, n} + e^{-j\pi/5\, n}}{2}\right)$$

$$= \underset{A}{10}\cos\left(\underset{B}{\pi/5}\, n + \underset{C}{0}\right)$$

$\{A\cos(Bn+C)\}$

3.13)

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt = \frac{\sin(4\omega)}{\omega}$$

$$x(t) = \begin{cases} 1 & 0 \le T < 4 \\ -1 & 4 \le T < 8 \end{cases}$$

$$T_0 = 8 \longrightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{8} = \pi/4$$

$$y(t) = ?$$

(i) Find a_k

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{8} \int_0^8 x(t) e^{-jk\pi/4 t} dt$$

$$= \frac{1}{8} \left[\int_0^4 (1) e^{-jk\pi/4 t} dt + \int_4^8 (-1) e^{-jk\pi/4 t} dt \right]$$

$$= \frac{1}{8} \left[\frac{e^{-jk\pi/4 t}}{-jk\pi/4} \Big|_0^4 - \left(\frac{e^{-jk\pi/4 t}}{jk\pi/4} \Big|_4^8 \right) \right]$$

$$= \frac{1}{8} \left[\left(\frac{e^{-jk\pi} - 1}{jk\pi/4} \right) - \left(\frac{1 - e^{-jk\pi}}{-jk\pi/4} \right) \right]$$

$$= \frac{1}{\cancel{8}_2} \cdot \frac{1}{-jk\pi/\cancel{4}} \left[2e^{-jk\pi} - 2 \right]$$

$$= \frac{1}{jk\pi} \left[1 - e^{-jk\pi} \right]$$

$$\therefore \; a_k = \begin{cases} 0 & k = 0, \pm 2, \pm 4, \ldots \\ \dfrac{2}{j\pi k} & k = \pm 1, \pm 3, \pm 5, \ldots \end{cases}$$

(ii) <u>Block Diagram</u>

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \xrightarrow{LTI} y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} H(jk\omega_0)$$

where;

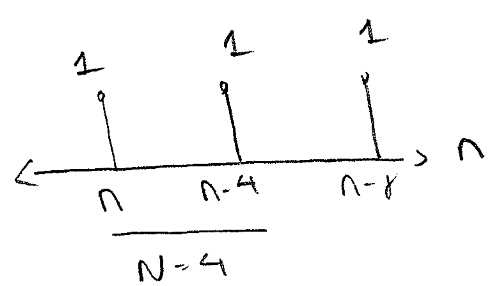
$$H(jk\omega_0) = H(jk\pi/4) = \frac{\sin(k\pi)}{k(\pi/4)}$$

(iii) Evaluate $H(jk\omega_0)$ at odd values of 'K'

$$\therefore \; y(t) = 0.$$ Ans:

3.14) $$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

$N = 4$



$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} e^{-jk\omega_0 n} x[n]$$

$$= \frac{1}{4} \sum_{n=0}^{3} x[n] e^{-jk 2\pi/4 \, n}$$

$$= \frac{1}{4}[1] = 1/4 \quad \text{for all } k.$$

$$y[n] = \sum_{k=0}^{3} a_k e^{+jk\frac{2\pi}{N}n} H(e^{j\frac{2\pi}{4}k})$$

$$= \frac{1}{4} H(e^{j0}) e^{j0} + \frac{1}{4} H(e^{j\pi/2}) e^{j\pi/2\, n} +$$

$$\frac{1}{4} H(e^{j3\pi/2}) e^{j3\pi/2\, n} + \frac{1}{4} H(e^{j\cancel{4}\pi}) e^{j\cancel{2}\pi n} \rightarrow (1)$$

$$y[n] = \cos\left(\frac{5\pi}{2} n + \pi/4\right) = \cos\left(\pi/2\, n + \pi/4\right)$$

$$= \frac{1}{2} e^{j(\pi/2\, n + \pi/4)} + \frac{1}{2} e^{-j(\pi/2\, n + \pi/4)}$$

$$= \frac{1}{2} e^{j(\pi/2\, n + \pi/4)} + \frac{1}{2} e^{j(3\pi/2\, n - \pi/4)} \rightarrow (2)$$

Compare (1) w/ (2);

$$\left.\begin{aligned} H(e^{j0}) &= H(e^{j\pi}) = 0 \\ H(e^{j\pi/2}) &= 2e^{j\pi/4} \\ H(e^{j3\pi/2}) &= 2e^{-j\pi/4} \end{aligned}\right\} \text{Ans.}$$

3.20) (a) $y(t) = V_c(t) \longrightarrow (1)$

i.e. $i_c(t) = C\,d/dt\, V_c(t) = C\,d/dt\, y(t)$

$i_c(t) = i_R(t) = i_L(t)$

$$i_R(t) = \frac{V_R(t)}{R}$$

$RC\,d/dt\, y(t) = V_R(t) \longrightarrow (2)$

$V_L(t) = LC\, d^2/dt^2\, y(t) \longrightarrow (3)$

$\therefore\ x(t) = LC\, d^2/dt^2\, y(t) + RC\, d/dt\, y(t) + y(t)$

$= d^2/dt^2\, y(t) + d/dt\, y(t) + y(t) \longrightarrow (4)$

(b) $x(t) = e^{j\omega t}$

$y(t) = H(j\omega)\, e^{j\omega t} \longrightarrow (5)$

Substituting (5) in (4);

$$e^{j\omega t} = H(j\omega)\left[d^2/dt^2\, e^{j\omega t} + d/dt\, e^{j\omega t} + e^{j\omega t} \right]$$

$$H(j\omega) = \frac{e^{j\omega t}}{e^{j\omega t}(-\omega^2 + j\omega + 1)} = \frac{1}{-\omega^2 + j\omega + 1}$$

(c) $x(t) = \sin t$ $\qquad T_0 = 2\pi \Longleftrightarrow \omega_0 = \frac{2\pi}{2\pi} = 1$

$$= \frac{1}{2j}\left(e^{j\,2\pi/2\pi\, t} - e^{-j\,2\pi/2\pi\, t} \right)$$

i.e. $a_1 = 1/2j$

$$a_{-1} = -\frac{1}{2j}$$

i.e. $a_1 = a^*_{-1} = \frac{1}{2j}$

$$\therefore\; y(t) = a_1 H(j\omega) e^{j\omega t} - a_{-1} H(-j\omega) e^{-j\omega t}$$

$$= \frac{1}{2j} H(j) e^{jt} - \frac{1}{2j} H(-j) e^{-jt}$$

$$= \frac{1}{2j} \left[H(j) e^{jt} - H(-j) e^{-jt} \right]$$

$$H[j(1)] = \frac{1}{-(1)^2 + j(1) + 1} = \frac{1}{j} \qquad H[j(-1)] = \frac{1}{-(-1)^2 + j(-1) + 1} = \frac{-1}{j}$$

$$= \frac{1}{2j} \left[\frac{e^{jt}}{j} + \frac{e^{-jt}}{j} \right]$$

$$= -\frac{1}{2} \left(e^{jt} + e^{-jt} \right) = -\cos t. \quad \text{Ans.}$$

3.28) (a) (i) $N = 7$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk2\pi/N\, n}$$

$$= \frac{1}{N} \sum_{n=0}^{4} (1) e^{-jk2\pi/7\, n} = \frac{1}{7} \sum_{n=0}^{4} e^{-jk2\pi/7\, n}$$

$=$

4.1.) (b) $e^{-2|t-1|}$

$$|t-1| = \begin{cases} (t-1) & t \geq 1 \\ -(t-1) & t < 1 \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{1} e^{+2(+t-1)} e^{-j\omega t}\, dt + \int_{1}^{\infty} e^{-2(t-1)} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{1} e^{2t} e^{-2} e^{-j\omega t}\, dt + \int_{1}^{\infty} e^{-2t} e^{2} e^{-j\omega t}\, dt$$

$$= \cancel{e^2}\left[e^{-2} \int_{-\infty}^{1} e^{t(2-j\omega)}\, dt + \int_{1}^{\infty} e^{-t(2+j\omega)}\, dt \right]$$

$$= \cancel{e^2}\left[e^{-2} \left\{ \frac{e^{t(2-j\omega)}}{2-j\omega} \Bigg|_{-\infty}^{1} \right\} + \left\{ \frac{e^{-t(2+j\omega)}}{-(2+j\omega)} \Bigg|_{1}^{\infty} \right\} \right]$$

$$= \cancel{e^2}\left[e^{-2} \left\{ \frac{e^{2} e^{-j\omega}}{2-j\omega} \right\} + e^{2} \left\{ \frac{\cancel{-}\, e^{-2} e^{-j\omega}}{\cancel{-}(2+j\omega)} \right\} \right]$$

$$= \frac{e^{-j\omega}}{2-j\omega} + \frac{e^{-j\omega}}{2+j\omega}$$

$$= e^{-j\omega}\left[\frac{1}{2-j\omega} + \frac{1}{2+j\omega} \right]$$

$$= \frac{4e^{-j\omega}}{4+\omega^2}$$ Ans.

4.2) (b). $x(t) = \frac{d}{dt}\{u(-2-t) + u(t-2)\}$

$$= \delta(t-2) - \delta(t+2)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

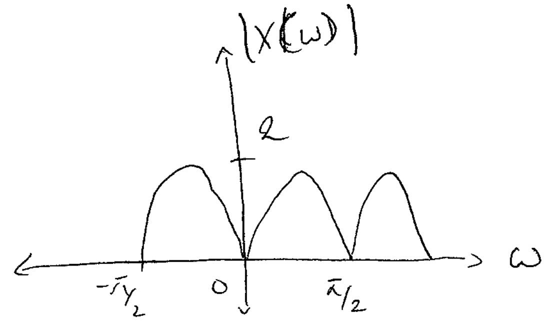
$$= \int_{-\infty}^{\infty} \delta(t-2) e^{-j\omega t} dt - \int_{-\infty}^{\infty} \delta(t+2) e^{-j\omega t} dt$$

$$= e^{-j\omega t}\Big|_{t=2} - e^{-j\omega t}\Big|_{t=-2}$$

$$= e^{-j\omega 2} - e^{j\omega 2}$$

$$= \frac{-2j}{2j}\left(e^{j\omega 2} - e^{-j\omega 2}\right)$$

$$= -2j\sin(2\omega) \quad \text{Ans:}$$



4.3.(b) $x(t) = 1 + \cos(6\pi t + \pi/8)$

$$= 1 + \frac{1}{2}e^{j\pi/8}e^{j6\pi t} + \frac{1}{2}e^{-j\pi/8}e^{-j6\pi t}$$

$$a_0 = 1$$

$$a_1 = \frac{1}{2}e^{j\pi/8}e^{j6\pi t}$$

$$a_{-1} = \frac{1}{2}e^{-j\pi/8}e^{-j6\pi t}$$

$$X(j\omega) = \sum_{k=-\infty}^{\infty} a_k 2\pi\delta(\omega - k\omega_0)$$ {The signal is periodic}

$$= a_0 2\pi\delta(\omega) + a_1 2\pi\delta(\omega-\omega_0) + a_{-1}2\pi\delta(\omega+\omega_0)$$

$$= 2\pi\delta(\omega) + e^{j\pi/8}(\pi)\delta(\omega-6\pi) + \pi e^{-j\pi/8}\delta(\omega+6\pi).$$

4.21) (a) $x(t) = [e^{-\alpha t} \cos\omega_0 t]u(t)$

$$= \frac{1}{2} e^{-\alpha t} e^{j\omega_0 t} u(t) + \frac{1}{2} e^{-\alpha t} e^{-j\omega_0 t} u(t)$$

$$X_1(j\omega) = \frac{1}{2}\int_0^\infty e^{-\alpha t} e^{j\omega_0 t} e^{-j\omega t} dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t(\alpha - j\omega_0 + j\omega)} dt$$

$$= \frac{1}{2}\left[\frac{e^{-t(\alpha - j\omega_0 + j\omega)}}{-(\alpha - j\omega_0 + j\omega)}\right]_0^\infty$$

$$= \frac{1}{2(\alpha - j\omega_0 + j\omega)} \longrightarrow (1)$$

$$X_2(j\omega) = \frac{+1}{2(\alpha + j\omega_0 + j\omega)}$$

$$\therefore\ X(j\omega) = X_1(j\omega) + X_2(j\omega)$$

$$= \frac{1}{2}\left[\frac{1}{\alpha + j(\omega - \omega_0)} + \frac{1}{\alpha + j(\omega + \omega_0)}\right]$$

(b) $e^{-3|t|} \sin 2t = x(t)$

$$x_1(t) = e^{-3|t|} \qquad |t| = \begin{cases} t & t \geq 0 \\ -t & t < 0 \end{cases}$$

$$X_1(j\omega) = \int_{-\infty}^0 e^{(3 - j\omega)t} dt + \int_0^\infty e^{-t(3 + j\omega)} dt$$

$$= \frac{e^{(3-j\omega)t}}{3-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-t(3+j\omega)}}{-(3+j\omega)}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{3-j\omega} + \frac{1}{3+j\omega} = \underline{\underline{\frac{6}{9+\omega^2}}}$$

$$x_2(t) = \sin 2t = \frac{1}{2j}\left[e^{j2t} - e^{-j2t}\right]$$

doesn't apply because the signal is periodic.

$$\left.\begin{aligned}
X_2(j\omega) &= \int_{-\infty}^{\infty} \sin 2t \, e^{-j\omega t}\, dt \\
&= \frac{1}{2j}\left[\int_{-\infty}^{\infty} e^{j2t} e^{-j\omega t}\, dt - \int_{-\infty}^{\infty} e^{-j2t} e^{-j\omega t}\, dt\right] \\
&= \frac{1}{2j}\left[\int_{-\infty}^{\infty} e^{t(j2-j\omega)}\, dt - \int_{-\infty}^{\infty} e^{-t(j2+j\omega)}\, dt\right] \\
&= \frac{1}{2j}\left[\frac{e^{t(j2-j\omega)}}{j(2-\omega)}\Bigg|_{-\infty}^{\infty} - \frac{e^{-t(j2+j\omega)}}{-(j2+j\omega)}\Bigg|_{-\infty}^{\infty}\right]
\end{aligned}\right\}$$

$$X_2(j\omega) = \frac{1}{2j}\left[2\pi\,\delta(\omega-2) - 2\pi\,\delta(\omega+2)\right]$$

$$= \frac{\pi}{j}\left[\delta(\omega-2) - \delta(\omega+2)\right]$$

$$\therefore\ X(j\omega) = \frac{1}{2\pi}\left[\left\{\frac{\pi}{j}\,\delta(\omega-2) - \delta(\omega+2)\right\} * \frac{6^{3}}{9+\omega^2}\right]$$

$$= -3j\left[\frac{1}{9+(\omega-2)^2} - \frac{1}{9+(\omega+2)^2}\right]$$

Ans.

4.21) (e) $x(t) = [te^{-2t} \sin 4t]u(t)$

$$X_1(j\omega) = \int_0^\infty te^{-2t} e^{-j\omega t}\,dt$$
$$= \int_0^\infty te^{-t(2+j\omega)}\,dt$$

d/dt	$\int$	$+/-$
t	$e^{-t(2+j\omega)}$	$+$
1	$\dfrac{e^{-t(2+j\omega)}}{-(2+j\omega)}$	$-$
0	$\dfrac{e^{-t(2+j\omega)}}{(2+j\omega)^2}$	$+$

$$= \frac{te^{-t(2+j\omega)}}{-(2+j\omega)} - \frac{e^{-t(2+j\omega)}}{(2+j\omega)^2}\Bigg]_0^\infty$$

$$= \frac{1}{(2+j\omega)^2}$$

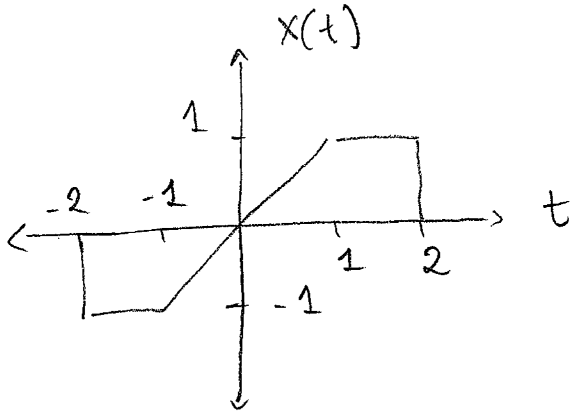
~~$X_2(j\omega)$~~ $x_2(t) = \sin 4t$
$$= \frac{1}{2j}\left[e^{j4t} - e^{-j4t}\right]$$
$$X_2(j\omega) = \frac{\pi}{j}\left[\delta(\omega-4) - \delta(\omega+4)\right]$$

$$\therefore \quad X(j\omega) = \left[X_1(j\omega) * X_2(j\omega)\right]\frac{1}{2\pi}$$

$$= \frac{1}{2\cancel{\pi}}\left[\frac{\cancel{\pi}}{j}\left\{\delta(\omega-4) - \delta(\omega+4)\right\} * \frac{1}{(2+j\omega)^2}\right]$$

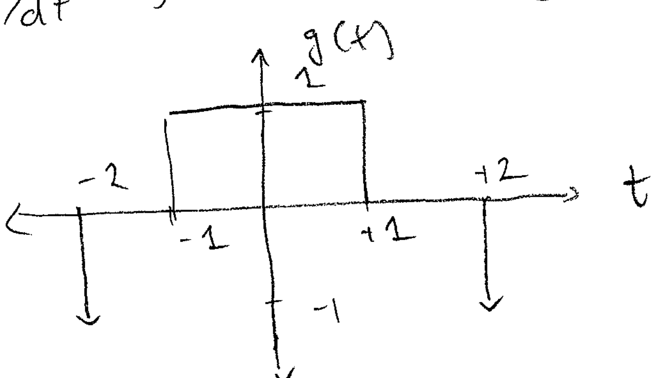
$$= \frac{1}{2j}\left[\frac{1}{[2+j(\omega-4)]^2} - \frac{1}{[2+j(\omega+4)]^2}\right]$$

4.21 (g)



$$x(t) \xrightarrow{F.T.} X(\omega)$$

$$g(t) = \frac{d}{dt}x(t) \xrightarrow{F.T.} G(\omega)$$



$$G_1(\omega) = -1\int_{-\infty}^{\infty} \delta(t+2)\, e^{-j\omega t}\, dt$$

$$= -e^{-j\omega t}\Big|_{t=-2}$$

$$= -e^{j\omega 2}$$

$$G_2(\omega) = \int_{-1}^{1} (1) e^{-j\omega t} dt$$

$$= \frac{e^{-j\omega t}}{-j\omega} \Big|_{\cancel{0}\,-1}^{1}$$

$$= \frac{e^{-j\omega}}{-j\omega} - \frac{e^{j\omega}}{-j\omega}$$

$$= \frac{1}{-j\omega} \left[e^{-j\omega} - e^{j\omega} \right]$$

$$= \frac{1}{j\omega} \left[e^{j\omega} - e^{-j\omega} \right]$$

$$= \frac{2}{\omega} \sin(\omega)$$

$$G_3(\omega) = -\int_{-\infty}^{\infty} \delta(t-2) e^{-j\omega t} dt$$

$$= -e^{-j\omega t} \Big|_{t=2} = -e^{-j\omega 2}$$

$$\therefore G(\omega) = G_1(\omega) + G_2(\omega) + G_3(\omega)$$

$$= -e^{j\omega 2} + \frac{2}{\omega}\sin(\omega) - e^{-j\omega 2}$$

$$= \frac{2\sin(\omega)}{\omega} - 2\cos(2\omega)$$

$$G(\omega) = j\omega X(\omega)$$

$$X(j\omega) = \left[\frac{j\omega}{G(\omega)}\right]^{-1} = \frac{2}{j\omega}\left[\frac{\sin(\omega)}{\omega} - \cos(2\omega)\right]$$