9 February 2012

Recall

0:	1
1:	1 1
2:	1 2 1
3:	1 3 3 1
4:	1 4 6 4 1
5:	1 5 10 10 5 1

$\Rightarrow$ sum of terms in row $i = 2^i$

$\Rightarrow$ alternating sum in row $i = 0$

↳ observation is obvious if i = odd (mirror values) symmetric

↳ why in even rows?

recall: sum of i^{th} row: explained by Binomial Theorem

$$(x+y)^n = \binom{n}{n}x^n + \binom{n}{n-1}x^{n-1}y + \dots + \binom{n}{n-i}x^{n-i}y^i \dots + \binom{n}{0}y^n$$

if $x, y = 1$, $(1+1)^n = \binom{n}{n} + \binom{n}{n-1} + \dots \binom{n}{n-i} + \dots \binom{n}{0}$

analogously:

if $x = 1, y = -1$: $(1-1^n) = \binom{n}{n}(1) + \binom{n}{n-1}(-1) + \dots (-1)^n\binom{n}{0}$

$= 0$ ✓

Q: $(x + \frac{1}{x})^{20} = x^{20} + \dots + \frac{1}{x^{20}}$

what is the coefficient of x^4? x^5?

A: consider $x = x$, $y = \frac{1}{x}$

applying binomial theorem:

$\Rightarrow$ typical term is $\binom{n}{n-i}x^{n-i}y^i = \binom{n}{n-i}x^{n-i}x^{-i} = \binom{n}{n-i}x^{n-2i}$

for x^4; $n=20$ $i=8$, so

$\binom{20}{20-8} = \binom{20}{12}x^4$ ✓

for x^5: ~~no~~ $n=20$ $i = 7\frac{1}{2} \notin \mathbb{N}$

no one of the terms

Q2: What is

$$2^0 \cdot 1 + 2^1 \cdot 5 + 2^2 \cdot 10 + 2^3 \cdot 10 + 2^4 \cdot 5 + 2^5 \cdot 1$$

A:

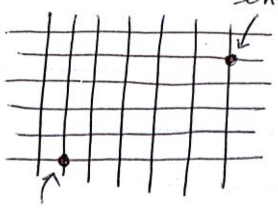
Letting $y = 2$, $x = 1$:

Binonomial Theorem

$$(1+2)^5 = \underbrace{\binom{5}{5}2^0 + \binom{5}{4}2^1}_{\text{coefficients of binomial tra}} + \dots$$

Q3: consider

school



home

In how many diff ways can you go from home to school w/o wasting time (optimal)

A: In any case, you would need to go east x times and North y times.

→ count the number of blocks to be walked E and N separately. and find the number of arrangement of E and N ("~~strings~~" e.g. EENENNE..")

→ Recall in the proof of binomial theorem, we established that $\binom{n}{n-i}$ = # of words of length n that has $n-i$ x's and i ys.

↳ numbers of walk involving $\{m \times E$ and $n \times N\}$

$$= \binom{m+n}{m} = \binom{m+n}{n}$$

Permutation w/ repetition

Q1: How ~~x~~ many "words" can you make by reordering the letters in the word "Mississippi".

Q2: How many fruit baskets can you make using apples, oranges and peaches, assuming that you want 10 fruits in each basket.

A2:

Make an algorithm that produces all possible fruit baskets of size 10, and the consider overcount.

Note. (1) Time of arrival of fruit is irrelvant

= think of recipient opening the fruit basket and counting contents

= interested in counting # of arangement of 10 letters each of the ~~each~~ either a or o or p.

Note2 In binomial theorem, we counteed # x_s ~~with~~ and y's but order was essential while supply of x_s and y_s was limited.

Since only content matters, we can simply count the number of words that look like:

$$\underbrace{aaa \ldots a\,ooo \ldots o \ldots opp \ldots p}_{n \text{ words long}} \quad \Rightarrow \text{order does not matter}$$

→ we only care about the transition of $a \longrightarrow o$ and $o \longrightarrow p$.
Then, we can deduce how the words look like.
= where 'a' ends and 'o' begins +
where 'o' ends and 'p' begins.

→ we can count arrangement of following

$$**|****|****$$

delimiters (identity of $*$ is not important)

Note

bars can be anywhere $*||**$ (a,pp) or $||**$ pp..

(my trial: consider 11^2

= there are 11 bar locations. However, this overcounts since

: $||$ (first, second) = $||$ (second, first)

A: $\displaystyle\sum_{i=0}^{10} \sum_{j=i}^{10} (1) = \binom{12}{2}$

we have 10 fruits and 2 separators: 12 symbols, 2 of which are special
⇒ $\binom{12}{2}$ arrangement of 2 types of symbols in which we have 2 and 10 respectively

What if we had baskets of size n using k fruit types?
$A = \binom{n+(k-1)}{k-1}$

Q: How many solution does

$$x_1 + x_2 + x_3 + x_4 = 24 \quad \text{if } x_i \in \mathbb{N}$$

A: Think of making a basket of size 24 with 4 kinds of fruits,
$= \binom{24+3}{3}$

imagine! 24 units of stuff, dole out to four people

Modification

(a) $x_1 + x_2 + x_3 + x_4 \le 24$

A $\sum_{i=0}^{24} \binom{i+3}{3}$? but this is pretty long calc. . .

or give 24 fruits out to 5 variables (dole 24 fruits to 5 kinds)

$\Rightarrow \binom{24+4+1-1}{4+1-1} = \binom{24+4+1-1}{4+1-1}$

(b) $x_1 + x_2 + x_3 + x_4 < 24$

$\Updownarrow$

≤ 23 (repeat) a.

(c) $x_1 + x_2 + x_3 + x_4 = 24, \quad x_2 \ge 7$

think of $x_1 + x_3 + x_4 \le 17$

or $= 17$ with 4 variables.

(d) $x_1 + x_2 + x_3 + x_4 = 24, \quad x_2 \le 7$

consider:

(1) All cases: $x_1 + x_3 + x_4 \le 24$

(2) Wrong cases: $x_1 + x_3 + x_4 \le 16 \quad (x_2 > 7)$

(3) (1) − (2) = good cases.

(e) $x_1 + x_2 + x_3 + x_4 = 24 \qquad 3 \le x_2 \le 17$

combine (c) and d.

→ reduce the problem to

$x_1 + x_2 + x_4 \le 21$

repead (d) for above with condition $x_2 \le 14$

$x_1 + x_2' + x_3 + x_4 = 21$

if $x_2' = x_2 - 3$

$3 \le x_2 \le 17; \quad 0 \le x_2 - 2 \le 14$

$= 0 \le x_2' \le 14$ ✓