ECE 302 Quiz 2
(10 points)

Name: Solution (3 points) 06/23/2016

1. Four students each flip a coin continually until the event A, defined as

$$A = \{\text{a majority of the students flip heads}\},$$

occurs for the first time. Assume that all flips are fair and independent. Find the probability that the event A occurs for the first time on their 5th trial. Show all work. (7 points)

$\Pr(A) = \Pr(\{\text{a majority of the students flip heads}\})$

$= \Pr(\{\text{3 heads in 4 flips}\} \cup \{\text{4 heads in 4 flips}\})$

Can use the Binomial Probability Law (or count)

$\Pr(A) = P_4(3) + P_4(4)$, with $\Pr(\{\text{heads}\}) = 1/2$

$= \binom{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 + \binom{4}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0$

$= 4 \cdot 1/16 + 1 \cdot 1/16 = 5/16$

Now use Geometric Probability Law

$\Pr(\{A \text{ occurs for the first time on } 5^{th} \text{ trial}\}) = p(5)$, with $\Pr(A) = 5/16$

$= (11/16)^4 (5/16)$