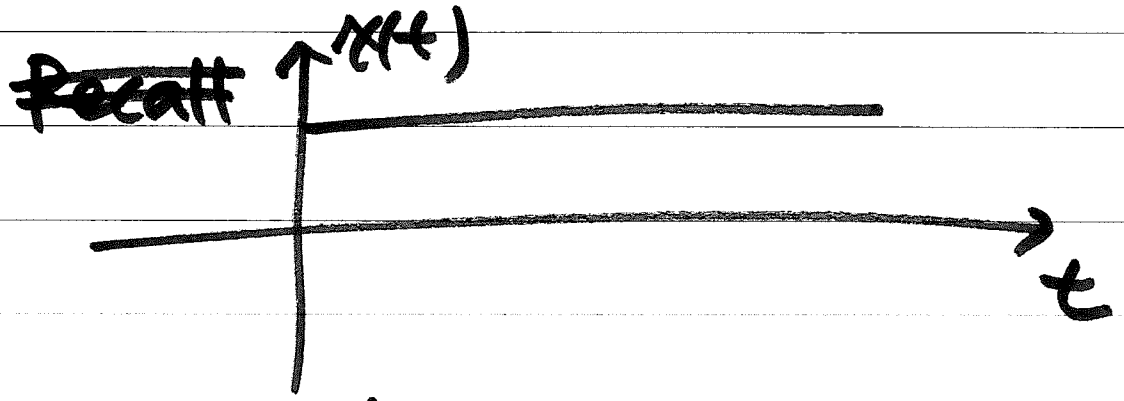
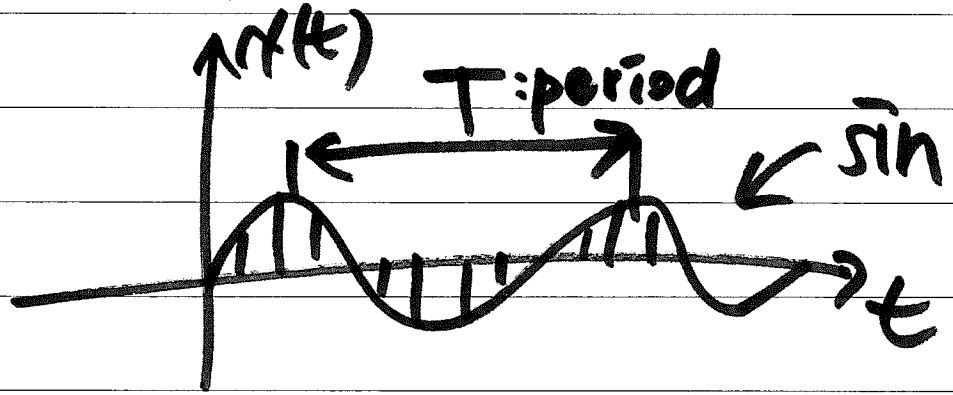
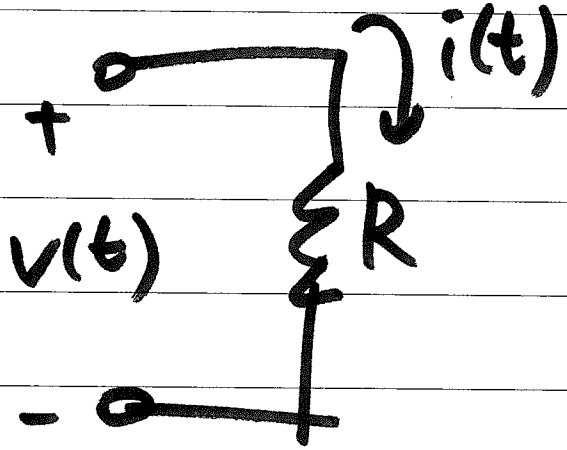
1-1. Signals

Energy · $E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$ · $x(t)$: signal





Recall:



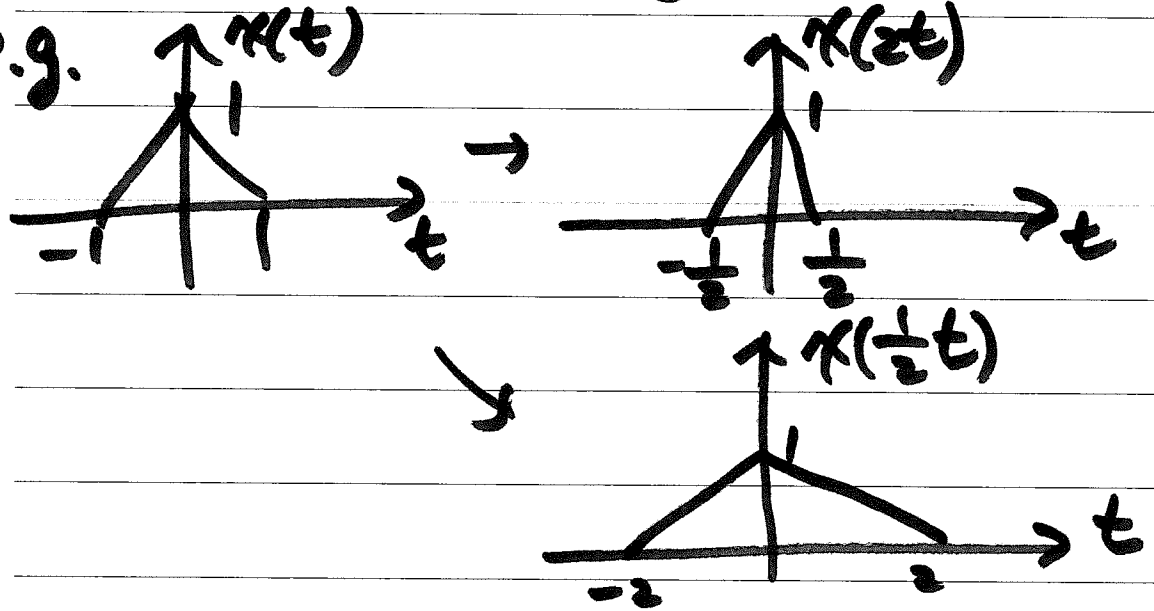
Energy dissipated in resistor:

$$\int_{-\infty}^{\infty} \frac{v^2(t)}{R}\,dt = \int_{-\infty}^{\infty} i^2(t)\,R\,dt$$

2) Transformation of Time Variable ($x(t)$)

- Time-shift : $x(t-t_0)$
- Time-Scaling : $x(at)$, $a>0$
  - $a>1$: compressed
  - $a<1$: expanded.

e.g.



- Time-Reversal : $x(-t)$

→ combination of the above

$x(at+b)$ or $x(\alpha(t-\beta))$

$x(at+b)$

$x(t)$

1. shift to left by b : $x(t+b)$
2. If $a<0$, flip about $t=0$ (time-reverse)
3. compress by a factor of $|a|$
   (if $|a|<1$: expansion) : $x(at+b)$

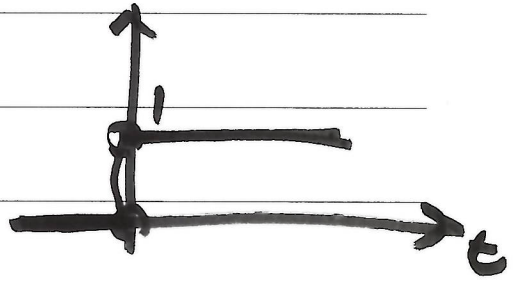
$x(\alpha(t-\beta))$

$x(t)$

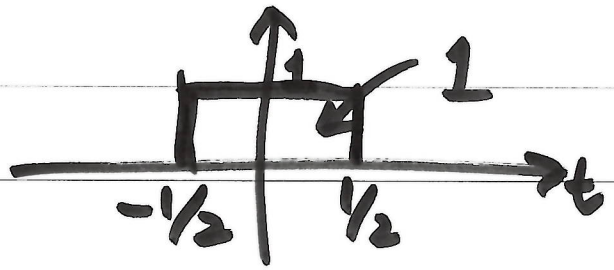
1. if $\alpha<0$, flip about $t=0$
2. if $|\alpha|>1$, compress by a factor of α
   (if $|\alpha|<1$: expansion) : $x(\alpha t)$
3. shift to right by β : $x(\alpha(t-\beta))$

3) Basic signals

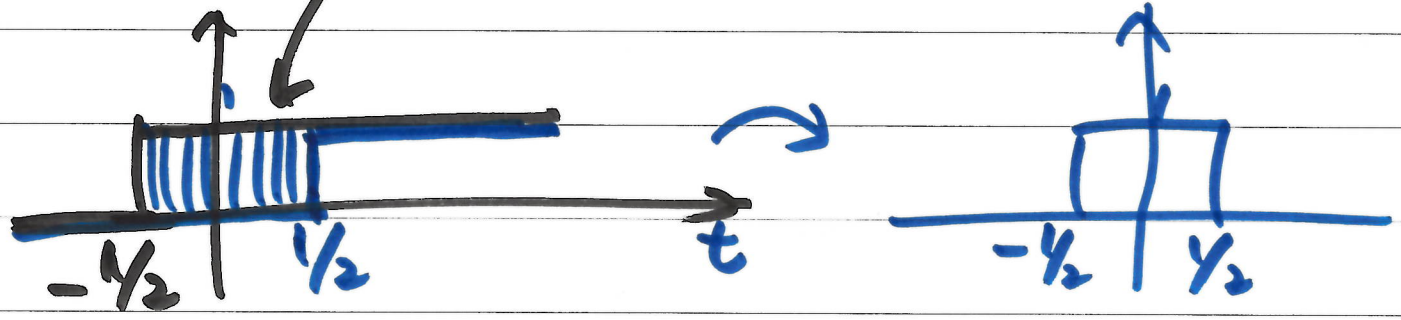
- unit step : $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$



- rect func. : $\text{rect}(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & |t| > 1/2 \end{cases}$



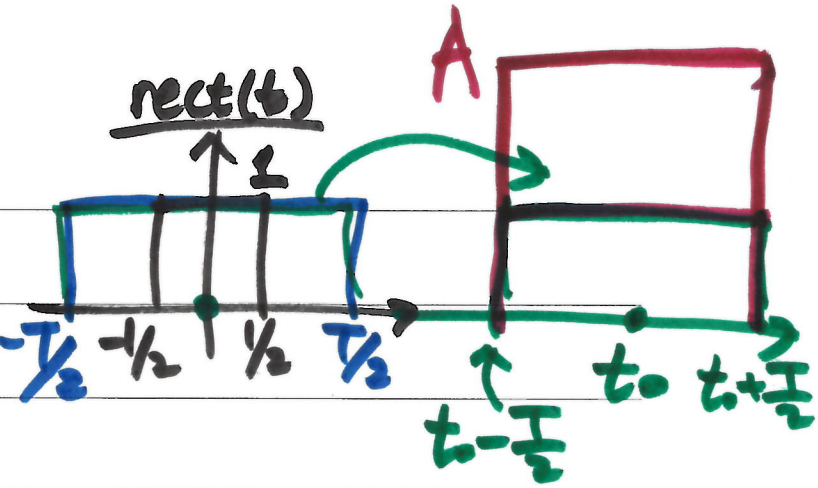
$\text{rect}(t) = u(t+1/2) - u(t-1/2)$



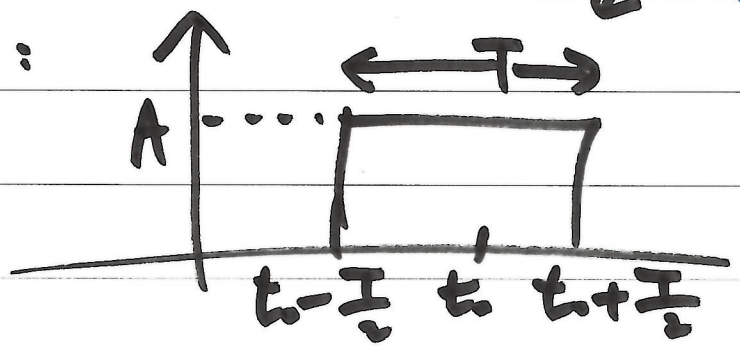
* rect : rectangle , func : function

6)

ex) $A\,\mathrm{rect}\left(\frac{t-t_0}{T}\right)$

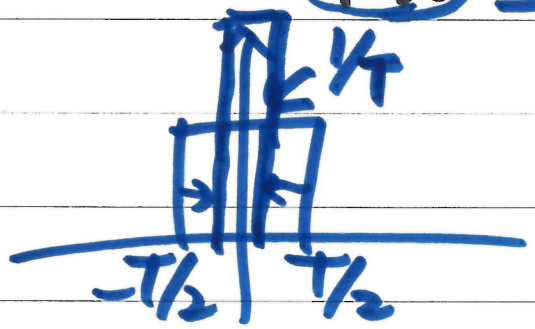


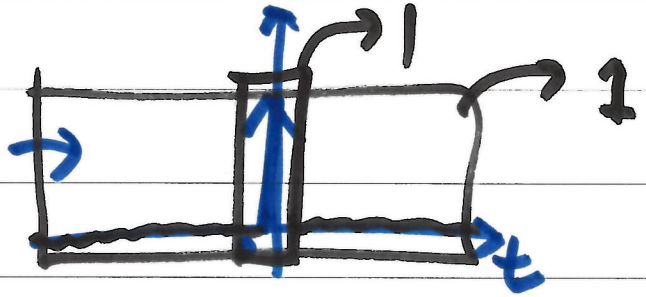
ans:



+) Dirac Delta Function

$$\delta(t) = \lim_{T\to 0} \frac{1}{T}\,\mathrm{rect}\left(\frac{t}{T}\right)$$



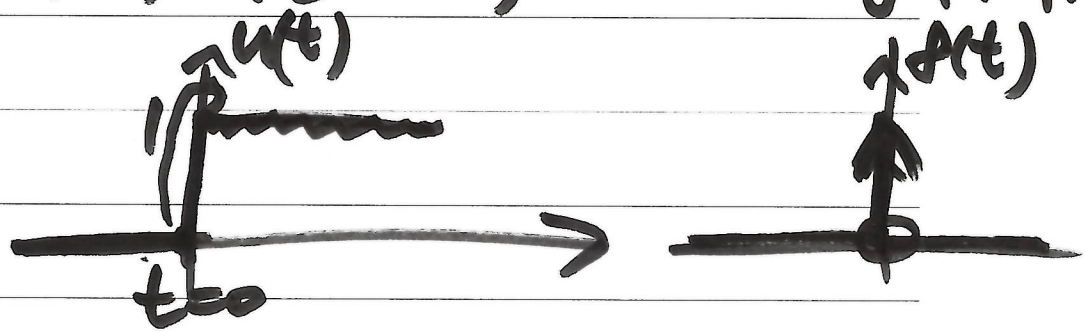


Properties of $\delta(t)$:

- $\int_{-\infty}^{\infty} \delta(t)\,dt = 1 = \int_{0^-}^{0^+} \delta(t)\,dt$

- $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$: shifting prop.

- $\delta(t) = \boxed{\frac{d\,u(t)}{dt}}$



5) Sinewaves : $x(t) = e^{j\omega_0 t}$

$(\omega_0 = 2\pi f_0)$, f_0 is freq. in Hz → (cycles per sec)

period : $e^{j\omega_0(t+T)} = e^{j\omega_0 t} = e^{j\omega_0 t} \cdot e^{j2\pi}$ (Period; $e^{j2\pi} = 1$)

$\rightarrow e^{j\omega_0 T} = e^{j2\pi} \rightarrow \omega_0 T = 2\pi \rightarrow \omega_0 = \frac{2\pi}{T}$ $(2\pi f_0)$

* freq : frequency $\rightarrow f_0 = \frac{1}{T}$

- By Euler's Formula.

$$\cos(\omega_0 t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

$$\sin(\omega_0 t) = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t} \quad //$$