1. Let $a_n, b_n \in \mathbb{R}$. Is it true that $\limsup a_n b_n = \limsup a_n \limsup b_n$? What if a_n and b_n are non-negative? Can you prove or disprove any inequality?

2. Rudin's definition of the limit supremum is

$$\limsup a_n = \sup E$$

with E the set of limit points of $\{a_n\}$. Show this is equivalent to the following definition

$$\limsup a_n = \lim_{n\to\infty} \sup_{k\geq n} a_k.$$

Hint: $\sup_{k\geq n} a_k$ decreasing in n. Remark: This second definition is often easier in practice.

3. Suppose X is a complete metric space, and $\{G_n\}$ is a family of open sets, and each G_n is dense, i.e. $\bar{G_n} = X$. Show $\cap G_n$ is nonempty.