(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No
If $y(t) = x(2t)$, is the system causal?		X
If $y(t) = (t+2)x(t)$, is the system causal?		X
choose $t = -\frac{1}{2}$ If $y(t) = x(-t^2)$, is the system causal?	X	
If $y(t) = x(t) + t - 1$, is the system memoryless?	X	
If $y(t) = x(t^2)$, is the system memoryless?		X
If $y(t) = x(t/3)$, is the system stable?	X	
If $y(t) = tx(t/3)$, is the system stable?		X
If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable?		X
If $y(t) = \sin(x(t))$, is the system time invariant?		X
If $y(t) = u(t) * x(t)$, is the system LTI?	X	
If $y(t) = (tu(t)) * x(t)$, is the system linear?		X

b/c convolution is linear

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(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-\tau}\underbrace{u(\tau)}\underbrace{u(t+2-\tau)}d\tau$$

$$u(\tau) = 0 \text{ for } \tau < 0 \Rightarrow y(t) = \int_0^{\infty} e^{-\tau}\underbrace{u(t+2-\tau)}d\tau$$

$$u(t+2-\tau) = 0 \text{ for } \tau > t+2 \Rightarrow y(t) = \int_0^{t+2} e^{-\tau}d\tau \leftarrow u(t+2)$$

false,

$$y(t) = -e^{-\tau}\Big]_0^{t+2} = -(e^{-(t+2)} - e^0)$$

$$\boxed{y(t) = 1 - e^{-(t+2)}}$$

NO

$\cancel{P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T}}$

$\boxed{2}$

(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$.

$$x(t) = \frac{3e^{jt}}{1+j}$$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left|\frac{3e^{jt}}{1+j}\right|^2 dt \quad \frac{3}{1+j}(e^{jt}) \Rightarrow \frac{3}{1+j}(\cos t + j\sin t)$$

$$\Rightarrow \frac{3(1-j)}{(1+j)(1-j)}(\cos t + j\sin t) = \frac{3-3j}{2}(\cos t + j\sin t) \quad \text{take Real}$$

$$\Rightarrow \frac{3}{2}\cos t + \frac{3}{2}\sin t$$

NO

$$E_\infty = \int_{-\infty}^{\infty} \left|\frac{3}{2}(\cos t \cancel{+} \sin t)\right|^2 dt = \frac{9}{4}\int_{-\infty}^{\infty} (1) dt$$

$$\cancel{P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T}} = \lim_{T\to\infty} \frac{\frac{9}{4}}{2T} = \lim_{T\to\infty} \frac{9}{8T} = \frac{9}{8(\infty)} = 0$$

$$\boxed{E_\infty = \frac{9}{4} \qquad P_\infty = 0}$$

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(15 pts) **4.** Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$

$$W_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$$

4 ak's?

(Simplify your answer as much as possible.)

$$\sin(\pi t) \Rightarrow \frac{e^{j\pi t} - e^{-j\pi t}}{2j} = \left(\frac{1}{2j}e^{j\pi t} - \frac{1}{2j}e^{-j\pi t}\right)$$

$$a_K = \frac{1}{4}\int_0^4 x(t)e^{-jk(\frac{\pi}{2})t}dt = \frac{1}{4}\left(\int_0^2 (\sin(\pi t))e^{-jk\frac{\pi}{2}t}dt + \int_2^4 \overbrace{0e^{-jk\frac{\pi}{2}t}}^{0}dt\right.$$

$$a_K = \frac{1}{4}\int_0^2\left(\frac{1}{2j}e^{j\pi t} - \frac{1}{2j}e^{-j\pi t}\right)e^{-jk\frac{\pi}{2}t}dt = \frac{1}{8j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right)e^{-jk\frac{\pi}{2}t}dt$$

$$a_0 = \frac{1}{8j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right)e^0 dt = \frac{1}{8j}\left(\frac{1}{\pi}(e^{j2\pi} - 1) + (e^{j2\pi} - 1)\right)$$

$$\boxed{a_0 = \frac{1}{4\pi j}(e^{j2\pi} - 1)}$$

$$a_1 = \frac{1}{8j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right)e^{-j\frac{\pi}{2}t}dt = \frac{1}{8j}\int_0^2 \left(e^{j\frac{\pi}{2}t} - e^{-j\frac{3\pi}{2}t}\right)dt$$

$$a_1 = \frac{1}{8j}\left(\left(\frac{2}{\pi}(e^{j\pi} - 1)\right) - \left(-\frac{2}{3\pi}(e^{-j3\pi} - 1)\right)\right) = \frac{1}{8j}\left(\frac{2}{\pi}e^{j\pi} - \frac{2}{\pi} + \frac{2}{3\pi}e^{-j3\pi} - \frac{2}{3\pi}\right)$$

$$\boxed{a_1 = \frac{1}{8j}\left(\frac{2}{\pi}e^{j\pi} + \frac{2}{3\pi}e^{-j3\pi} + \frac{4}{3\pi}\right)}$$

$$a_{-1} = \frac{1}{8j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right)e^{+j\frac{\pi}{2}t}dt = \frac{1}{8j}\int_0^2 \left(e^{j\frac{3}{2}\pi t} - e^{-j\frac{\pi}{2}t}\right)dt$$

$$\boxed{a_{-1} = \frac{1}{8j}\left(\frac{2}{3\pi}e^{3\pi} + \frac{2}{\pi}e^{j\pi} - \frac{4}{3\pi}\right)}$$

$$a_2 = \frac{1}{8j}\int_0^2 \left(e^{j\pi t} - e^{-j\pi t}\right)e^{-j\pi t}dt = \frac{1}{8j}\int_0^2 \left(0 - e^{-j2\pi t}\right)dt = \frac{1}{8j}\left(-\frac{1}{2}(e^{-j4\pi} - 1)\right)$$

$$\boxed{a_2 = -\frac{1}{16j}(e^{-j4\pi} - 1)}$$

5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

$$
\begin{array}{rcl}
\text{input} & & \text{output} \\
x_0[n] = \delta[n] & \rightarrow & y_0[n] = \delta[n-1], \\
x_1[n] = \delta[n-1] & \rightarrow & y_1[n] = 4\delta[n-2], \\
x_2[n] = \delta[n-2] & \rightarrow & y_2[n] = 9\delta[n-3], \\
x_3[n] = \delta[n-3] & \rightarrow & y_3[n] = 16\delta[n-4], \\
 & \vdots & \\
x_k[n] = \delta[n-k] & \rightarrow & y_k[n] = (k+1)^2\delta[n-(k+1)] \text{ for any integer k.}
\end{array}
$$

(10 pts) **a)** Can this system be time-invariant? Explain.

0

$x_2[n-n_0] \stackrel{?}{=} \delta[n-n_0-2] \stackrel{?}{\Rightarrow} y_2[n-n_0] \stackrel{?}{=} 9\delta[n-n_0-3]$

Let $n_0=3$

$x_2[n-3] \stackrel{?}{=} \delta[n-5] \stackrel{?}{\Rightarrow} y_2[n-3] \stackrel{?}{=} 9\delta[n-5]$

original $x_2[n]$ equation required $n=2$ for $y_2[n]\neq 0$ b/c of $\delta[n-2]$, with $n_0=3$, n now has to $=5$ for $y_2[n]\neq 0$. So $y_2[5-3]=y_2[2]$ yes TI.

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?

$u[n-1] = \int_{-\infty}^{\infty} \delta[\tau-1]\,d\tau$

$u[n-1] = 0$ for $n<1$

$= 1$ for $n \geq 1$

0

$x_0[n]=\delta[n] \Rightarrow y_0[n]=\delta[n-1]$

$\boxed{x[n] = u[n]}$:(