ECE 302 Homework 7
Due August 2, 2016

Reading assignment: chapter 9, sections 9.1 - 9.4, 9.6; chapter 10, sections 10.1 - 10.3.

1. A coin is flipped n times. Let the random variable $X_i = 1$ if the ith flip is heads and $X_i = 0$ if the ith flip is tails, for $i = 1, 2, ..., n$. Let X be the number of heads flipped in n flips. Assume all flips are fair and independent.

   (a) What kind of random variable is X? Express X as a function of $X_1, X_2, ..., X_n$.

   (b) Find the mean and variance of X.

2. A random process $X(t)$ is defined by

$$X(t) = \begin{cases} 1 & , T \leq t \leq T+1, \\ 0 & , \text{else}, \end{cases}$$

   where T is a uniformly distributed random variable in the interval (0,1).

   (a) Plot a few sample functions of $X(t)$.

   (b) Find the pmf of $X(t)$ for a fixed value of t.

   (c) Find $\mu_X(t)$ and $R_X(t_1, t_2)$. Is $X(t)$ a wide-sense stationary random process?

3. A discrete-time random process is defined by $X(n) = A^n$, for $n \geq 0$. Assume A is a uniform random variable on the interval $(0, 1)$.

   (a) Plot a few sample functions of $X(n)$.

   (b) Find the pdf of $X(n)$ for a fixed value of n.

   (c) Find $\mu_X(n)$ and $R_X(n_1, n_2)$. Is $X(n)$ a wide-sense stationary random process?

4. Students arrive at a train station according to a Poisson process with an arrival rate of 1 student per 5 minutes.

   (a) Find the probability that the first student will arrive in the first 10 minutes.

   (b) Find the probability that the first two students will arrive in the first 10 minutes.

   (c) Find the probability that no more than two students will arrive in the first 10 minutes.

5. The input into a filter is zero-mean white Gaussian noise X(t) with noise power density $N_0/2$ W/Hz. The filter has transfer function

$$H(f) = \frac{1}{1 + j2\pi f}.$$

   (a) Find $R_X(\tau)$.

   (b) The process $X(t)$ is sampled at two time points $t_1 \neq t_2$, yielding $X(t_1)$ and $X(t_2)$. Are $X(t_1)$ and $X(t_2)$ uncorrelated?

   (c) Let $Y(t)$ be the output of the filter. Find $S_Y(f)$ and $R_Y(\tau)$. What is the average power of $Y(t)$?

   (d) Find the average power of $Y(t)$ in the frequency range $[-10, 10]Hz$.