Problem 1 $x(t) \longleftrightarrow X(j\omega)$

(a) $x^*(t) \longleftrightarrow X^*(-j\omega)$ directly from Table. If we desire to prove directly

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

conjugate

$$x^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(j\omega)\, e^{-j\omega t}\, d\omega$$

c.o.v.
$\omega \mapsto -\omega$
$d\omega \mapsto -d\omega$

$$= \frac{1}{2\pi}\int_{+\infty}^{-\infty} X^*(-j\omega)\, e^{+j\omega t}\, (-d\omega)$$

flip limits

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(-j\omega)\, e^{+j\omega t}\, d\omega$$

(b) $\mathrm{Re}\{x(t)\}$

Note $\mathrm{Re}\{x(t)\} = \dfrac{x(t) + x^*(t)}{2}$

$\updownarrow$

Fourier transform $= \dfrac{X(j\omega) + X^*(-j\omega)}{2}$

(c) $x(t-2) + x^*(-t-2)$

$x(t-2) \longleftrightarrow e^{-j2\omega} X(j\omega)$ from Table

$$x^*(t-2) \longleftrightarrow \left[e^{-j2\omega} X(j\omega)\right]^*\Big|_{j\omega \mapsto -j\omega}$$

$$= e^{+j2\omega} X^*(j\omega)\Big|_{j\omega \mapsto -j\omega}$$

$$= e^{-j2\omega} X^*(-j\omega)$$

$$x^*(-t-2) \longleftrightarrow \left[e^{-j2\omega} X^*(-j\omega)\right]\Big|_{j\omega \mapsto -j\omega}$$

$$= e^{+j2\omega} X^*(j\omega) \quad \text{from Table entries}$$

$$\therefore \; x(t-2) + x^*(-t-2) \longleftrightarrow \underbrace{e^{-j2\omega} X(j\omega) + e^{+j2\omega} X^*(j\omega)}$$

Can also write as $X(j\omega) = |X(j\omega)|\, e^{j\angle X(j\omega)}$

$$|X(j\omega)| \left\{ e^{j(2\omega - \angle X(j\omega))} + e^{-j(2\omega - \angle X(j\omega))} \right\}$$

$$= 2|X(j\omega)| \cos\left(2\omega - \angle X(j\omega)\right).$$

(d) $e^{j4\pi t} x(t/3)$

$$x(t/3) \longleftrightarrow 3X(j3\omega)$$

$$e^{j4\pi t} x(t/3) \longleftrightarrow 3X(j3(\omega - 4\pi)).$$

Problem 2

$$BW = \underbrace{\frac{1}{|H(j0)|^2}}_{\text{this from transform definition}} \; \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} |H(j\omega)|^2 \, d\omega}_{\text{this from Parseval}}$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt \;\Rightarrow\; H(j0) = \int_{-\infty}^{\infty} h(t)\, dt$$

$$= 2 - 1 + 1 - 1$$

$$= \underline{1}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} |H(j\omega)|^2 \, d\omega = \int_{-\infty}^{\infty} |h(t)|^2 \, dt$$

$$= 4 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1$$

$$= 7$$

$$\therefore \; BW = 7/\underline{1} = 7.$$

Problem 3

(a) $z_I(t) = x_I(t)\cos\omega_c t$

$$\Updownarrow$$

$$Z_I(j\omega) = \frac{1}{2\pi} X_I(j\omega) * \pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

$$= \frac{1}{2} X_I(j(\omega-\omega_c)) + \frac{1}{2} X_I(j(\omega+\omega_c))$$

$z_Q(t) = x_Q(t)\sin\omega_c t$

$$\Updownarrow$$

$$Z_Q(j\omega) = \frac{1}{2\pi} X_Q(j\omega) * \frac{\pi}{j}\left[\delta(\omega-\omega_c) - \delta(\omega+\omega_c)\right]$$

$$= -\frac{j}{2} X_Q(j(\omega-\omega_c)) + \frac{j}{2} X_Q(j(\omega+\omega_c))$$

Since $X_I(j\omega)$ and $X_Q(j\omega)$ are assumed to be real-valued see that

$Z_I(j\omega)$ is pure real

$Z_Q(j\omega)$ is pure imaginary

$$Z(j\omega) = Z_I(j\omega) + Z_Q(j\omega)$$

$$= \underbrace{\left[\frac{X_I(j(\omega-\omega_c)) + X_I(j(\omega+\omega_c))}{2}\right]}_{\mathrm{Re}\{Z(j\omega)\}} + j\underbrace{\left[\frac{-X_Q(j(\omega-\omega_c)) + X_Q(j(\omega+\omega_c))}{2}\right]}_{\mathrm{Im}\{Z(j\omega)\}}$$

(b) Since $h_c(t) = \delta(t)$ here also have $H_c(j\omega) = 1$
Therefore $z_c(t) = z(t)$.

$$s_I(t) = z(t)\, 2\cos\omega_c t$$

$$\updownarrow$$

$$S_I(j\omega) = \frac{1}{2\pi} Z(j\omega) * 2\pi\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right]$$

$$= Z(j(\omega-\omega_c)) + Z(j(\omega+\omega_c))$$ ← this is enough to plot using part (a).

(if want to write in terms of X_I and X_Q)

$$= \left[\frac{1}{2} X_I(j(\omega-2\omega_c)) + X_I(j\omega) + \frac{1}{2} X_I(j(\omega+2\omega_c))\right]$$

$$+ j\left[-\frac{1}{2} X_Q(j(\omega-2\omega_c)) + \frac{1}{2} X_Q(j(\omega+2\omega_c))\right]$$

$$s_Q(t) = z(t)\, 2\sin\omega_c t$$

$$\updownarrow$$

$$S_Q(j\omega) = \frac{1}{2\pi} Z(j\omega) * \frac{2\pi}{j}\left[\delta(\omega-\omega_c) - \delta(\omega+\omega_c)\right]$$

$$= -jZ(j(\omega-\omega_c)) + jZ(j(\omega+\omega_c))$$ ← enough to plot using (a)

(in terms of X_I and X_Q)

$$= \left[-\frac{1}{2} X_Q(j(\omega-2\omega_c)) + X_Q(j\omega) - \frac{1}{2} X_Q(j(\omega+2\omega_c))\right]$$

$$+ j\left[-\frac{1}{2} X_I(j(\omega-2\omega_c)) + \frac{1}{2} X_I(j(\omega+2\omega_c))\right]$$

(c) The LP filters will select on the frequencies $|\omega| \le \omega_M$ in both $S_I(j\omega)$ and $S_Q(j\omega)$.

Note from picture that the imaginary parts are zero for $|\omega| \le \omega_M$.

$$\therefore \quad Y_I(j\omega) = H_{LP}(j\omega)\, S_I(j\omega) = X_I(j\omega)$$

$$Y_Q(j\omega) = H_{LP}(j\omega)\, S_Q(j\omega) = X_Q(j\omega).$$

(d) Now assume a general channel filter $H_c(j\omega)$.

Then $Z_c(j\omega) = H_c(j\omega) Z(j\omega)$ and looking only at the in-phase branch

$$Y_I(j\omega) = H_{LP}(j\omega)\, S_I(j\omega)$$

$$= H_{LP}(j\omega)\left[Z_c(j(\omega-\omega_c)) + Z_c(j(\omega+\omega_c))\right]$$

$$= H_{LP}(j\omega)\left[H_c(j(\omega-\omega_c)) Z(j(\omega-\omega_c)) + H_c(j(\omega+\omega_c)) Z(j(\omega+\omega_c))\right]$$

$$[\;\;] = H_c(j(\omega-\omega_c))\left\{\tfrac{1}{2} X_I(j(\omega-2\omega_c)) + \tfrac{1}{2} X_I(j\omega) - \tfrac{j}{2} X_Q(j(\omega-2\omega_c)) + \tfrac{j}{2} X_Q(j\omega)\right\}$$

$$+ H_c(j(\omega+\omega_c))\left\{\tfrac{1}{2} X_I(j\omega) + \tfrac{1}{2} X_I(j(\omega+2\omega_c)) - \tfrac{j}{2} X_Q(j\omega) + \tfrac{j}{2} X_Q(j(\omega+2\omega_c))\right\}.$$

Now apply $H_{LP}(j\omega)$ to $[\;]$ above will remove the spectral terms centered at $\pm 2\omega_c$:

$$H_{LP}(j\omega)[\;]$$

$$= H_{LP}(j\omega)\,H_c(j(\omega-\omega_c))\left\{\tfrac{1}{2}X_I(j\omega) + \tfrac{j}{2}X_Q(j\omega)\right\}$$

$$+ H_{LP}(j\omega)\,H_c(j(\omega+\omega_c))\left\{\tfrac{1}{2}X_I(j\omega) - \tfrac{j}{2}X_Q(j\omega)\right\}$$

$$= \tfrac{1}{2}X_I(j\omega)\,H_{LP}(j\omega)\left[H_c(j(\omega-\omega_c)) + H_c(j(\omega+\omega_c))\right]$$

$$+ \tfrac{j}{2}X_Q(j\omega)\,H_{LP}(j\omega)\left[H_c(j(\omega-\omega_c)) - H_c(j(\omega+\omega_c))\right]$$

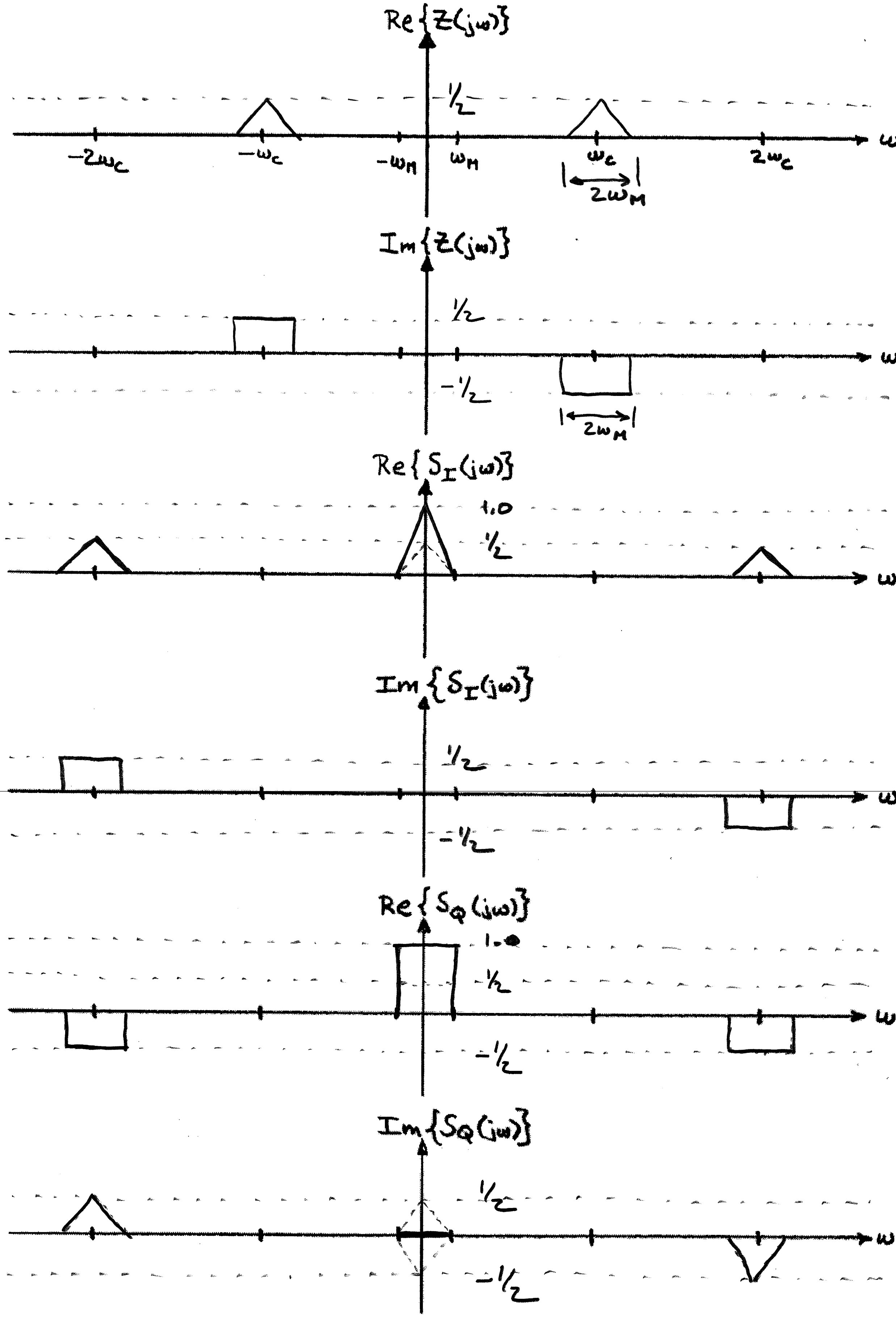
This must be zero to have no cross talk. Since channel filter has a real-valued imp. response $H_c(j\omega) = H_c^*(-j\omega)$

$$\Longrightarrow H_c(j(\omega-\omega_c)) = H_c^*(j(\omega_c-\omega))$$

$\therefore$ Suff. cond. for no cross talk

$$H_{LP}(j\omega)\left[H_c^*(j(\omega_c-\omega)) - H_c(j(\omega_c+\omega))\right] = 0$$

$\forall \omega$.



Re{Z(jω)}
1/2
ω
−2ωc
−ωc
−ωM
ωM
ωc
2ωc
2ωM
Im{Z(jω)}
1/2
−1/2
ω
2ωM
Re{SI(jω)}
1.0
1/2
ω
Im{SI(jω)}
1/2
−1/2
ω
Re{SQ(jω)}
1.0
1/2
−1/2
ω
Im{SQ(jω)}
1/2
−1/2
ω