5 April

Platonic Solids

Def: A platonic solid is a 3-D solid that

→ is bounded

→ a boundary made up of a finite number of plane polygons (a round cup is not platonic)

→ all boundary polygons have the same number of vertices)

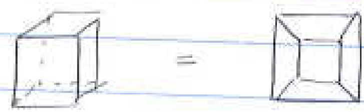
→ ~~are~~ is regular (i.e. all sides have same length, all angle same size)

→ at the vertices the number of incoming edges is always the same

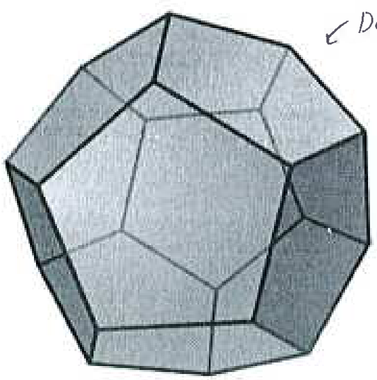
→ at any vertex, for any two neighboring edges the angle between them is always the same

Ex

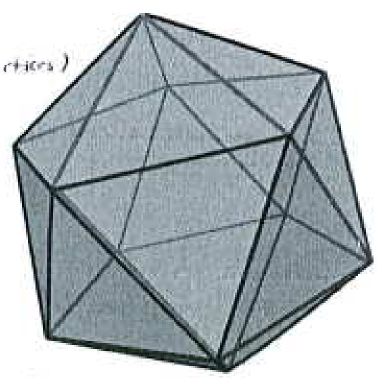
Cube



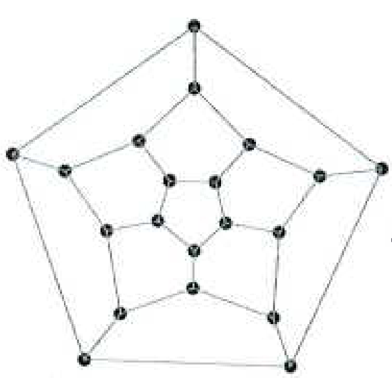
" view of cube if we stare inside the cube very close to a face "



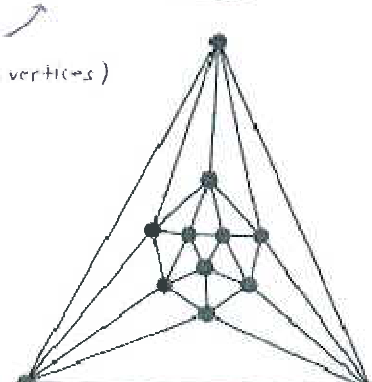
Dodecahedron
(12 faces, 30 edges, 20 vertices)



icosahedron
(20 faces, 30 edges, 12 vertices)



P.S.
Professor expects us to reproduce these images . .



Goal! Prove that this is a complete list of platonic solids.

We see that the edge graphs of platonic solids are
- → connected → vert w/ same degree : d
- → planar → faces w/ same degree : D
- → simple

Let e = # of edges
f = # of faces

Recall Handshake Lemma for...

vertices :
$$2e = \sum_{verts} \deg(\text{vertex}) = vd$$

faces
$$2e = \sum_{face} \deg(\text{face}) = fD$$

using Euler's Equation (connected graphs) ... we rid of v & f.

$$e + 2 = v + f$$
$$= dDe + dD2 = dDv + dDf$$
$$= dDe + 2dD = 2eD + 2ed$$
$$= 2dD = e(2d + 2D - dD)$$
$$2 = e\left(\frac{2}{D} + \frac{2}{d} - 1\right)$$

We note that e, D, d are all natural numbers (= eq above is solvable)
this puts several restriction on the equation we found

Conside
$\frac{2}{D} + \frac{2}{d} - 1$. cannot be negative.
↳ this is big constraint. Most choices for $d, D \in \mathbb{N}$ will render it negative.

So, Question becomes
$$\frac{2}{D} + \frac{2}{d} > 1$$

we also remind ourselves that

$d=1$ would not make any sense

moreover, degree must be ≥ 3 for it to connect completely w/ other vertices (3 incoming edges) n 3D)

$D=1$ would be a loop

2 would be a mult edge

≥ 3.

tabulating possibilities.

$n \backslash d$	3	4	5	6	7
3	$\frac{1}{3}$	+	+	0	−
4	+	0	−	−	−
5	+	−	−	−	−
6	−	−	−	−	−
7					

$\frac{2}{d} + \frac{2}{D} - 1$

Five positives

Five choices.

thus

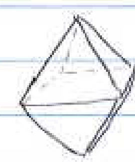
$d = D = 3$: tetrahedron

$d=3, D=4$: cube

$d=3, D=5$: dodecahedron

$d=5, D=3$: icosahedron

$d=4, D=3$: octahedron



the list of platonic solids.

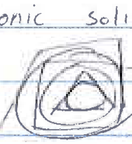
Math Stories:

Greek discovered the same five geometric solids.

corresponded nicely with the five known planets!

~1000yrs later: Kepler & Copernicus.

if platonic solids are drawn



in a inscribing fashion, the relative size of the solids corresponds to the inner radius of planetary orbits

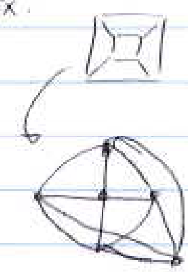
Silly Discoveries: permutation of five solids. one permutation could coincidentally agree w/ ratios radius

Theory/beauty marred by subsequent discoveries of other planets.

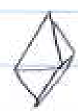
Dual Graphs

Suppose G is a planar graph. Pick a plane representation.

Ex.



Def the dual graph of G has
- → as vertices faces of the given graph G
- → two vertices linked if the faces were neighbors in G

$\Longleftrightarrow$  octahedron

Dual graph of octahedron, likewise, is a cube.

Under reasonable condition, dual of a dual is the original.

Challenge: Find G planar w/ $(G^{\perp})^{\perp} \neq G$.

$$(\text{icosahedron})^{\perp} = \text{dodecahedron}$$
$$(\triangle^{\perp}) = \text{itself}$$

Note

$$V_{G^{\perp}} = f \qquad e_G = e_{G^{\perp}}$$
$$V_G = f_{G^{\perp}}$$