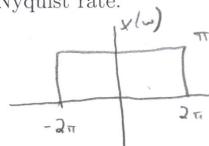


1. Determine whether each of the following signals is band limited. (Answer yes/no and justify.) If they are band limited, specify their Nyquist rate.

$$(5 \text{ pts}) \text{ a) } x_1(t) = \frac{\sin(2\pi t)}{t}$$

$$\frac{\sin \omega t}{\pi t} \xrightarrow{\text{FT}} \frac{1}{\pi} (\omega + \omega) - \omega (\omega - \omega)$$

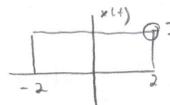
$$x_1(t) \xrightarrow{\text{FT}} \frac{1}{\pi} (\omega + 2\pi) - \omega (\omega - 2\pi)$$



Band limited

$$Nyquist = 4\pi$$

$$(5 \text{ pts}) \text{ b) } x_2(t) = 3(u(t+2) - u(t-2))$$



Not band limited

Sharp corners and finite fine

$$(5 \text{ pts}) \text{ c) } x_3(t) = \frac{\sin(2\pi t)}{t} \sum_{n=-\infty}^{\infty} \delta(t - \frac{n}{2})$$

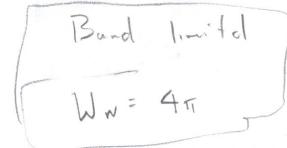
use Multiplication property

$$x(t) = \frac{\sin(2\pi t)}{t}$$

$$y(t) = \sum_{n=-\infty}^{\infty} \delta(t - \frac{n}{2})$$

$$x(t) y(t) \xrightarrow{\text{FT}} \frac{1}{2\pi} X(\omega) * Y(\omega)$$

$$= \frac{1}{2\pi} \left((\pi u(\omega + 2\pi) - \omega (\omega - 2\pi)) * \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \delta(\omega - \frac{n}{2}) \right)$$



(5 pts) d) $x_4(t) = \left(\frac{\sin(2\pi t)}{t}\right)^2$
 Use multiplication + then expand again
 $X_1(t) = \frac{\sin 2\pi t}{t} \quad X_2(t) = \frac{\sin(2\pi t)}{t}$

Yes band limited

$$\omega_n = 2(4\pi) = 8\pi$$

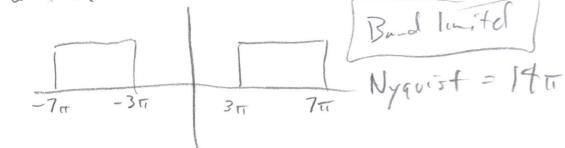
(5 pts) d) $x_5(t) = \frac{\sin(2\pi t)}{t} \cos(5\pi t)$
 $\cos 5\pi t = \frac{e^{j5\pi t}}{2} + \frac{e^{-j5\pi t}}{2}$
 $X_s(t) = \frac{e^{j5\pi t}}{2} \frac{\sin(2\pi t)}{2} + \frac{e^{-j5\pi t}}{2} \frac{\sin(2\pi t)}{2}$

Use Freq. Shift property:

$$X_s(\omega) = \frac{\pi}{2} (v(\omega + 2\pi - 5\pi) + v(\omega - 2\pi - 5\pi) + v(\omega + 2\pi + 5\pi) + v(\omega - 2\pi + 5\pi))$$

$$(5 \text{ pts}) e) x_6(t) = \frac{\sin(2\pi t)}{t} e^{j5\pi t} = \frac{\pi}{2} (v(\omega - 3\pi) - v(\omega - 7\pi) + v(\omega + 7\pi) - v(\omega + 3\pi))$$

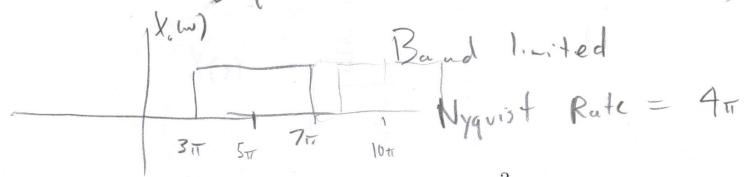
Freq. Shift fng



Band limited

$$\text{Nyquist} = 14\pi$$

$$X_s(\omega) = \frac{\pi}{2} (v(\omega + 2\pi - 5\pi) - v(\omega - 2\pi - 5\pi))$$



(15 pts) 2. Using the definition of the z-transform (i.e. do not simply take the answer from the table), obtain the z-transform (with its ROC) of

$$x[n] = 3^n u[-n-1]$$

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n] z^{-n} \\ &= \sum_{n=-\infty}^{\infty} 3^n u[-n-1] z^{-n} \end{aligned}$$

$$= \sum_{n=-\infty}^{-1} 3^n z^{-n}$$

$$= \sum_{n=-\infty}^{-1} \left(\frac{3}{z}\right)^n \quad m = n+1$$

$$= \sum_{m=0}^{\infty} \left(\frac{3}{z}\right)^{m-1}$$

$$= \frac{z}{3} \sum_{m=0}^{\infty} \left(\frac{3}{z}\right)^m$$

$$\boxed{X(z) = \frac{z}{3} \frac{1}{1 - \frac{3}{z}}}$$

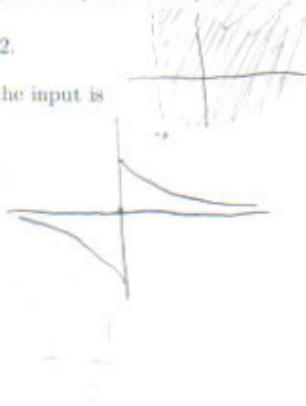
(20 pts) 3. The Laplace transform of the unit impulse response of a system is

$$H(s) = \frac{1}{s+2}, \operatorname{Re}(s) > -2.$$

Determine the response $y(t)$ of the system when the input is

$$x(t) = \begin{cases} e^{-3t} & t > 0 \\ 0 & t = 0 \\ -e^{3t} & t < 0 \end{cases}$$

$$\chi(t) = \begin{cases} 1 & t > 0 \\ -\frac{1}{e^3} & t = 0 \\ -1 & t < 0 \end{cases}$$



$$x(t) = e^{-2t} u(t) - e^{2t} u(-t)$$

$$Y(s) = \frac{1}{s+3} - \frac{1}{s-3} \quad \text{from } (s+2) \text{ and } (s-3)$$

$$= \frac{(s-3) - (s+3)}{(s+3)(s-3)} = -\frac{6}{(s+3)(s-3)}$$

$$Y(s) = H(s) \chi(s)$$

$$Y(s) = \frac{-6}{(s+3)(s-3)(s+2)} = \frac{A}{s+3} + \frac{B}{s-3} + \frac{C}{s+2}$$

$$A = -1$$

$$B = -1/5$$

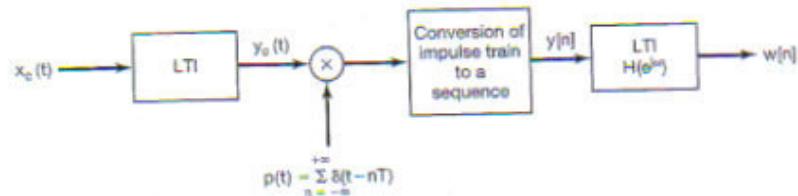
$$C = 6/5$$

$$y(t) = -e^{-t} u(t) - \frac{1}{5} e^{2t} u(-t) + \frac{6}{5} e^{-2t} u(t)$$

4. The block diagram below shows a system consisting of a continuous-time LTI system followed by a sampler, conversion to a sequence, and an LTI discrete-time system. The continuous-time LTI system is causal and satisfies the linear, constant-coefficient differential equation

$$\frac{dy_c(t)}{dt} + y_c(t) = x_c(t).$$

The input $x_c(t)$ is a unit impulse $\delta(t)$.



- (10 pts) a) Determine the input $y_c(t)$.

$$s y_c(s) + y_c(s) = X(s)$$

$$y_c(s) (1 + s) = X(s) = 1$$

$$y_c(s) = \frac{1}{1+s}$$

$$y_c(t) = e^{-t} u(t)$$

(Problem 7 continues on the next page.)

(15 pts) b) Determine the frequency response $H(e^{j\omega})$ and the unit impulse response $h[n]$ such that $w[n] = \delta[n]$.

$$y[n] \rightarrow H(\omega) \rightarrow w[n]$$

$$\omega[n] = s[n]$$

$$H(\omega) = \frac{w(\omega)}{y(\omega)} = \frac{1}{y(\omega)}$$

$$y[-\tau] = y_e(-\tau) = e^{-nT} v(-\tau)$$

$$x_1(\omega) = \frac{1}{1 - e^{-T} e^{-j\omega}} \text{ by (3.8)}$$

$$x_1(\omega) \geq e^{-T}$$

$$H(\omega) = \frac{1}{1 - e^{-T} e^{-j\omega}}$$

$$h[n] = s[n] - e^{-nT} s[n-1]$$