1. Countable or uncountable (with proof)?

   (a) $\oplus_{\mathbb{N}}\mathbb{Q} = \{(q_1, q_2 ...) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times ... :$ only finitely many q_i are non-zero.$\}$.

   (b) $\Pi_{\mathbb{N}}\mathbb{Q} = \{(q_1, q_2 ...) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times ...\}$

2. For any open $G \subseteq \mathbb{R}$, show G is a countable union of balls, i.e. $\exists\{B_{r_j}(x_j)\}_{j\in\mathbb{N}}$ with $G = \cup_j B_{r_j}(x_j)$.

3. Let F_j be closed subsets, and G_j open subsets of a metric space (X, d). Proof or counter-example:

   (a) $\cap_{j=1}^{\infty} G_j$ is open.

   (b) $\cup_{j=1}^{\infty} F_j$ is closed.

4. Consider $\mathbb{Z}$ the integers as a subspace of $\mathbb{R}$. Let $A \subseteq \mathbb{Z}$. Is A open in $\mathbb{Z}$ (i.e. is A open relative to $\mathbb{Z}$?) Is A open in $\mathbb{R}$. What about closed relative to $\mathbb{Z}$? $\mathbb{R}$?