# Midterm Examination 2 

ECE 302, Spring 2013
Instructor: Prof. Mimi Boutin

## Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. You have 50 minutes to complete this exam. You must remain sitting until the end of the exam is announced. You may not leave the exam early. When the end of the exam is announced, you must stop writing immediately. Anyone caught writing after the exam is over will get a grade of zero.
3. Do not tear out any page from this booklet.
4. You must keep your eyes on your desk at all times. Looking around is not allowed.
5. This is a closed book exam. The use of calculators is prohibited. Cell phones, iPods, and all other electronic communication devices are strictly forbidden. This means that they MUST BE TURNED OFF (not on vibrate mode) and stowed away (in your bag, not in your pocket) AT ALL TIMES.

Name: $\qquad$
Email: $\qquad$
Signature: $\qquad$

| Itemized Scores |
| :--- |
| Problem 1: |
| Problem 2: |
| Problem 3: |
| Problem 4: |
| Problem 5: |
| Total: |

(20 pts) 1. Let X be a one-dimensional normal random variable with mean equal to one and standard deviation equal to two. Use the standard normal table on page 7 to compute the following probabilities
a) $\operatorname{Prob}(1.02<X<1.08)$
b) $\operatorname{Prob}(0.98<X<1.18)$
(Give the numerical values for each probability and write the intermediate steps you followed to get these values.)
(25 pts) 2. Let X be a discrete random variable with probability mass function

$$
p_{X}(n)=\left\{\begin{array}{lc}
C \frac{1}{3^{n}}, & n=0,1,2,3, \ldots \\
0, & n=-1,-2,-3, \ldots
\end{array}\right.
$$

where C is a constant. What is the expected value of X ? (Justify your answer.) Hint: Recall the geometric series $\frac{1}{1-r}=\sum_{k=0}^{\infty} r^{k}$ for any $r$ with $|r|<1$.
(20 pts) 3. Let $X_{1}, X_{2}$ be two continuous random variables with pdf as follows:

$$
f_{X_{1}}(x)=f_{X_{2}}(x)= \begin{cases}1, & 0 \leq x \leq 1 \\ 0, & \text { else }\end{cases}
$$

Assume that $X_{1}$ and $X_{2}$ independent. What is the pdf of the random variable $Y=\max \left\{X_{1}, X_{2}\right\} ?$ (You must clearly justify your answer to get full credit.)
(25 pts) 4. A 2D continuous random variable $X=\left(X_{1}, X_{2}\right)$ is uniformly distributed on the triangle defined by the vertices $(0,0),(3,0)$, and $(0,4)$. Find
a) the marginal $f_{X_{1}}\left(x_{1}\right)$,
b) the conditional density $f_{X_{2} \mid X_{1}}\left(x_{2} \mid x_{1}\right)$.
(20 pts) 5. A random variables X has the following probability density function:

$$
f_{X}(x)=C e^{-\frac{1}{2} \frac{(x+3)^{2}}{9}},
$$

where C is a constant. What is the expected value of X? (Justify your answer mathematically.)


|  | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | 07 | 08 | 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | . 5000 | . 5040 | . 5080 | . 5120 | . 5160 | . 5199 | . 5239 | . 5279 | . 5319 | 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | . 5714 | 5753 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 | . 6103 | . 6141 |
| 0.3 | . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6406 | . 6443 | 6480 | 6517 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
| 0.5 | 15 | . 6950 | 6985 | . 7019 | 054 | 088 | 23 | 7 | . 7190 | 7224 |
| 6 | . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | 7454 | . 7486 | . 7517 | . 7549 |
| 7 | . 7580 | . 7611 | . 7642 | . 7673 | . 7704 | . 7734 | 7764 | . 7794 | . 7823 | . 7852 |
| 0.8 | . 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 8051 | . 8078 | . 8106 | . 8133 |
| 0.9 | . 8159 | . 8186 | . 8212 | . 823 | . 8264 | . 8289 | . 8315 | . 8340 | . 8365 | . 8389 |
| 1.0 | . 8413 | 838 | . 8461 | 85 | . 8508 | 31 | . 8554 | . 8577 | 8599 | 8621 |
| 1.1 | . 8643 | . 8665 | . 8686 | . 8708 | . 8729 | . 8749 | . 8770 | . 8790 | . 8810 | . 8830 |
| 1.2 | . 8849 | . 8869 | . 8888 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | . 9015 |
| 1.3 | . 9032 | . 9049 | . 9066 | . 9082 | . 9099 | . 9115 | . 9131 | . 9147 | . 9162 | . 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 | . 9306 | . 9319 |
| 1. | . 9332 | . 9345 | 557 | 70 | 882 | 94 | 406 | . 9418 | 429 | 441 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
| 1.7 | . 9554 | . 9564 | . 9573 | . 9582 | . 9591 | . 9599 | . 9608 | . 9616 | . 9625 | . 9633 |
| 1.8 | . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9686 | . 9693 | . 9699 | . 9706 |
| 1.9 | . 9713 | . 9719 | . 9726 | . 9732 | . 9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
| 2.0 | . 9772 | 78 | . 9783 | . 9788 | . 9793 | . 9798 | . 9803 | . 9808 | . 9812 | . 9817 |
| 2. | . 9821 | . 9826 | . 9830 | . 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 | . 9887 | . 9890 |
| 2.3 | . 9893 | . 9896 | . 9898 | . 9901 | . 9904 | . 9906 | . 9909 | . 9911 | . 9913 | . 9916 |
| 2.4 | . 99.18 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | . 9931 | . 9932 | . 9934 | . 9936 |
| 2.5 |  | . 9940 |  | . 9943 | . 9945 | 946 | 948 | 949 | 51 | 9952 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | . 9962 | . 9963 | . 9964 |
| 2.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | . 9971 | . 9972 | . 9973 | . 9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| 2.9 | . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 | . 9986 | . 9986 |
| 3.0 | . 9 | . 9987 | . 9987 | . 9988 | 988 | . 9989 | . 9989 | . 9989 | . 9990 | . 9990 |
| 3.1 | . 9990 | . 9991 | . 9991 | . 9991 | . 9992 | . 9992 | . 9992 | . 9992 | . 9993 | . 9993 |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
| 3.3 | . 9995 | . 9995 | . 9995 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9998 |

The standard normal table. The entries in this table provide the numerical values of $\Phi(y)=\mathbf{P}(Y \leq y)$, where $Y$ is a standard normal random variable, for $y$ between 0 and 3.49. For example, to find $\Phi(1.71)$, we look at the row corresponding to 1.7 and the column corresponding to 0.01 , so that $\Phi(1.71)=.9564$.
-SCRATCH -
(will not be graded)
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(will not be graded)

