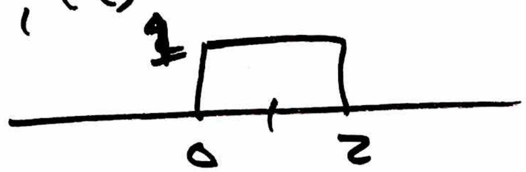
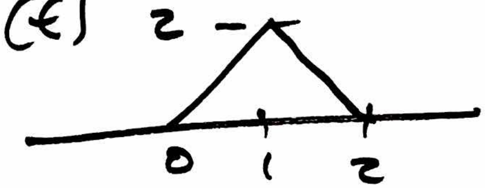
1.31

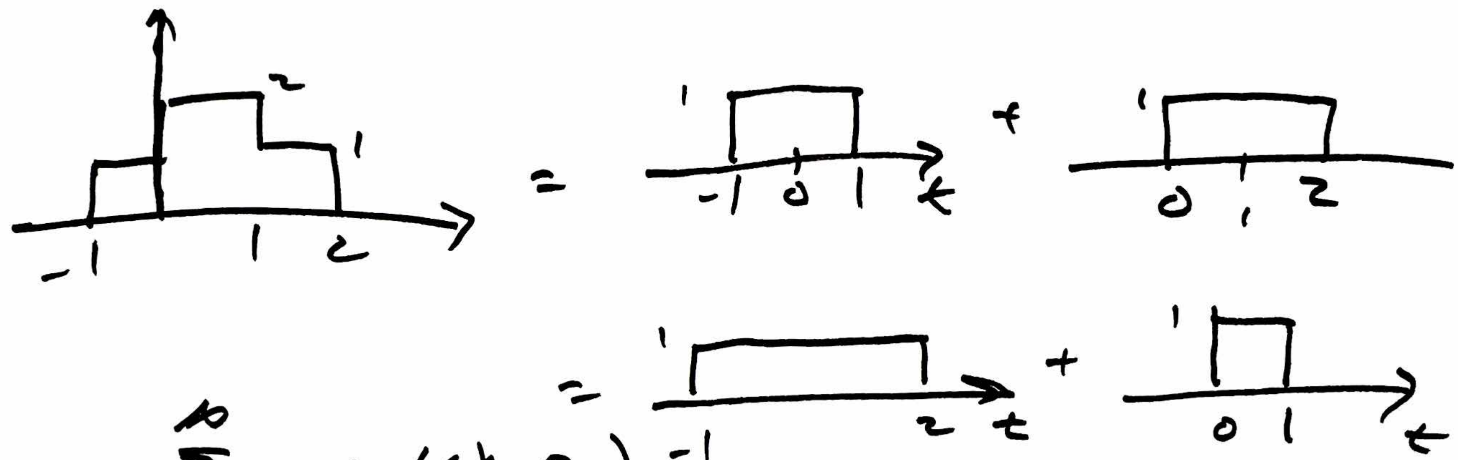
$x_1(t)$



$y_1(t)$



$x_3(t)$



$$x_3(t) = \sum_{k=0}^{\infty} a_k x_1(t - \tau_k)$$

$$S\{x_3(t)\} = S\{\sum_{k=0}^{\infty} a_k x_1(t-\tau_k)\}$$

$$= \sum_{k=0}^{\infty} a_k \underbrace{S\{x_1(t-\tau_k)\}}_{y_1(t-\tau_k)}$$

$$y(t) = x(t) * h(t)$$

$$y(\tfrac{t}{3}) = x(\tfrac{t}{3}) * h(\tfrac{t}{3})$$

$$\neq x(\tfrac{t}{3}) * h(t)$$

Time shift in $\rightarrow$ time shift out

amplitude scaled in $\rightarrow$ amplitude scaled out

time scaled in $\not\rightarrow$ time scaled out

■ DT systems described by constant coefficient difference equations

- Use difference eqn to describe <u>causal</u> <u>LTI</u> systems

<u>Ex</u>

$$y[n] = 2x[n] + x[n-1] + 4x[n-3]$$

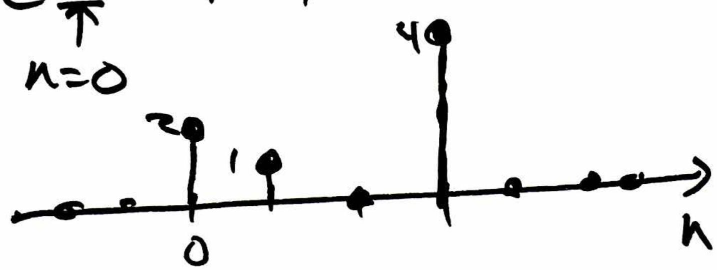
$y[n]$ depends only on $x[n-k]$

$$x[n] = \delta[n] \Rightarrow y[n] = h[n]$$

$$h[n] = 2\delta[n] + \delta[n-1] + 4\delta[n-3] \checkmark$$

$$= \{\underset{\substack{\uparrow \\ n=0}}{2}, 1, 0, 4\}$$

=



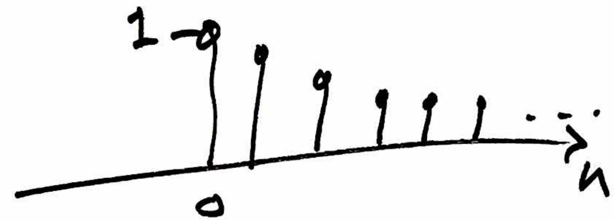
Ex $y[n] = 0.5\,y[n-1] + x[n]$

$x[n] = \delta[n] \Rightarrow y[n] = h[n]$

$h[n] = 0.5\,h[n-1] + \delta[n]$

n	$h[n]$	$h[n-1]$	$\delta[n]$
0	1	0	1
1	0.5	1	0
2	$(0.5)^2$	0.5	0
3	$(0.5)^3$	$(0.5)^2$	0
4	$(0.5)^4$	$(0.5)^3$	0
	⋮	⋮	

$h[n] = (0.5)^n u[n]$



$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$a_0 y[n] + \sum_{k=1}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = \frac{1}{a_0}\left[\sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]\right]$$

General form

$N=0 \quad y[n] = \frac{1}{a_0}\sum_{k=0}^{M} b_k x[n-k]$

$$x[n] = \delta[n] \Rightarrow y[n] = h[n]$$

$$h[n] = \frac{1}{a_0}\sum_{k=0}^{M} b_k \delta[n-k]$$

$$= \frac{1}{a_0} \cdot \{\underset{\underset{n=0}{\uparrow}}{\underline{b_0}}, b_1, b_2, \dots, b_M\}$$

Finite impulse response system/Filter

FIR

- Recursive equations can lead to an infinite impulse response (IIR) system/filter.

OW 2.5

$$x[n] = \begin{cases} 1 & 0 \le n \le 9 \\ 0 & \text{else} \end{cases}$$

$$h[n] = \begin{cases} 1 & 0 \le n \le N \\ 0 & \text{else} \end{cases}$$

$y[4] = 5$, $y[14] = 0$

should be $=$ ✗

In general, length$(x[n] * h[n])$ $\le$ length$(x[n])$ + length$(h[n])$ − 1

$10 + N + 1 - 1 \le 14$

← not given $y[13]$

$N \le 4$

$$y[4] = \sum_{k=0}^{4} h[4-k]\, x[k] = 5$$

$h[4], h[3], h[2], \ldots h[0] = 1$

$N \ge 4$

$N = 4$

Where did this come from? The length of the output must be $10 + (N+1) - 1$. Given that $y[14] = 0$, the length of the output must be less than or equal to 14.

Given length$(x[n])$ and length$(h[n])$ as N and M, there are exactly $N+M-1$ points of overlap between the sequences.

2.46) $x(t) = 2e^{-3t}u(t-1)$

$$x(t) \xrightarrow{S} y(t)$$

$$\frac{d}{dt}x(t) \xrightarrow{S} \overbrace{-3y(t) + e^{-2t}u(t)}^{y'(t)}$$

Find $h(t)$?

$$y'(t) = S\{x'(t)\} = x'(t) * h(t)$$

$$x'(t) = 2e^{-3}\delta(t-1) + 2(-3)e^{-3t}u(t-1)$$
$$= 2e^{-3}\delta(t-1) - 6e^{-3t}u(t-1)$$

$(f \cdot g)' = f' g + f \cdot g'$

$$x'(t) = \frac{d}{dt}(2e^{-3t})u(t-1) + 2e^{-3t}\frac{d}{dt}(u(t-1))$$
$$= -6e^{-3t}u(t-1) + \underbrace{2e^{-3t}\delta(t-1)}_{\text{sampling property}}$$
$$= -6e^{-3t}u(t-1) + 2e^{-3}\delta(t-1)$$

$$(x'(t) * h(t)) = \Big(\underbrace{-6e^{-3t}u(t-1)}_{-3x(t)} + 2e^{-3}\delta(t-1)\Big) * h(t)$$

$$= -3x(t) * h(t) + 2e^{-3}\,\underline{\delta(t-1) * h(t)}$$

$$= -3y(t) + 2e^{-3}h(t-1)$$

$$= -3y(t) + e^{-2t}u(t) \leftarrow \text{given information}$$

$$2e^{-3}h(t-1) = e^{-2t}u(t)$$

$$h(t-1) = \tfrac{1}{2}e^{-2t}e^{3}u(t)$$

$$= \tfrac{1}{2}e^{3}e^{-2t}u(t)$$

$$h(t) = \tfrac{1}{2}e^{3}e^{-2(t+1)}u(t+1)$$

Takeaway

$$y'(t) = x'(t) * h(t)$$

example

$$s(t) = u(t) * h(t)$$

$$h(t) = \frac{d}{dt}s(t)$$

$$\frac{d}{dt}u(t-\tau) = \delta(t-\tau)$$