ECE 301 Signals and Systems
Homework # 8 Solution

$\boxed{7.21}$ The sampling frequency is,

$$\omega_s = \frac{2\pi}{T} = 2\times10^4\pi$$

a) Yes. because $\omega_M = 5000\pi$. $\omega_s > 2\omega_M$.

b) No. because $\omega_M = 15000\pi$. $\omega_s < 2\omega_M$

c) No. because no information is given about the $Im\{X(j\omega)\}$, ω_M may be greater than $\frac{\omega_s}{2}$.

d) Yes, because $x(t)$ real $\Rightarrow$ $X(j\omega)$ even. $\Rightarrow |X(j\omega)| = 0$ $|\omega| > 5000\pi$.

$\omega_s > 2\omega_M$.

e) No. Similar argument as d), but in this case, $\omega_s < 2\omega_M$.

f) Yes. If $X(j\omega) * X(j\omega) = 0$ for $|\omega| > 15000\pi$,

then $X(j\omega) = 0$ for $|\omega| > 7500\pi$,

$\omega_M = 7500\pi$,

$\omega_s > 2\omega_M$.

g) Yes. $|X(j\omega)| = 0 \Rightarrow X(j\omega) = 0$.

$\omega_M = 5000\pi$.

$\omega_s > 2\omega_M$.

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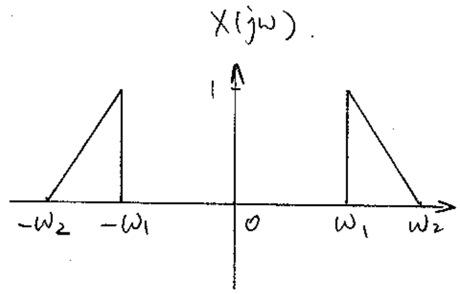
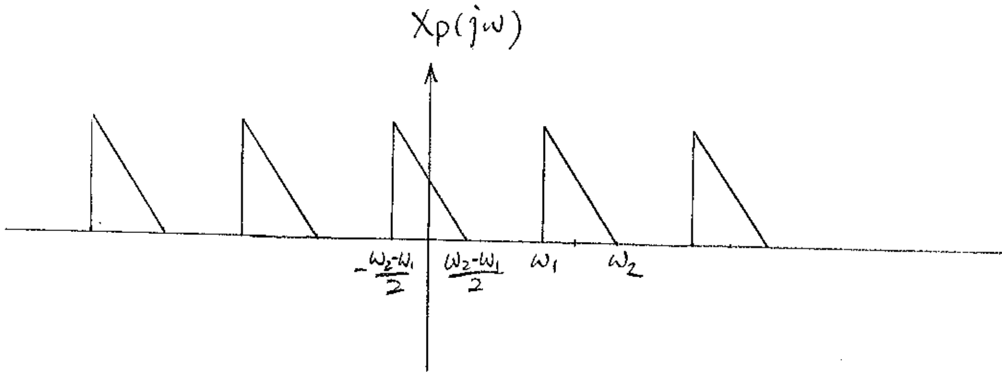
7.27

a)





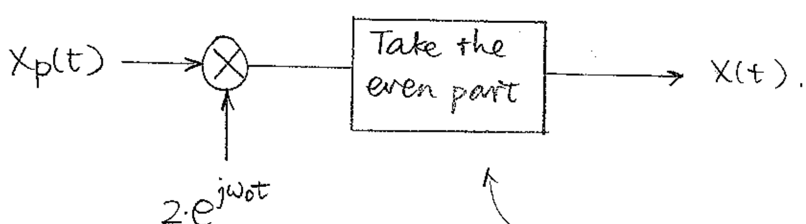
b) Here $\omega_M = \frac{\omega_2-\omega_1}{2}$.

So the maximum sampling period T is governed by the relationship

$$\frac{2\pi}{T} > 2\omega_M = \omega_2-\omega_1$$

$$T < \frac{\omega_2-\omega_1}{2\pi}$$

c)



Note that this system is not LTI.

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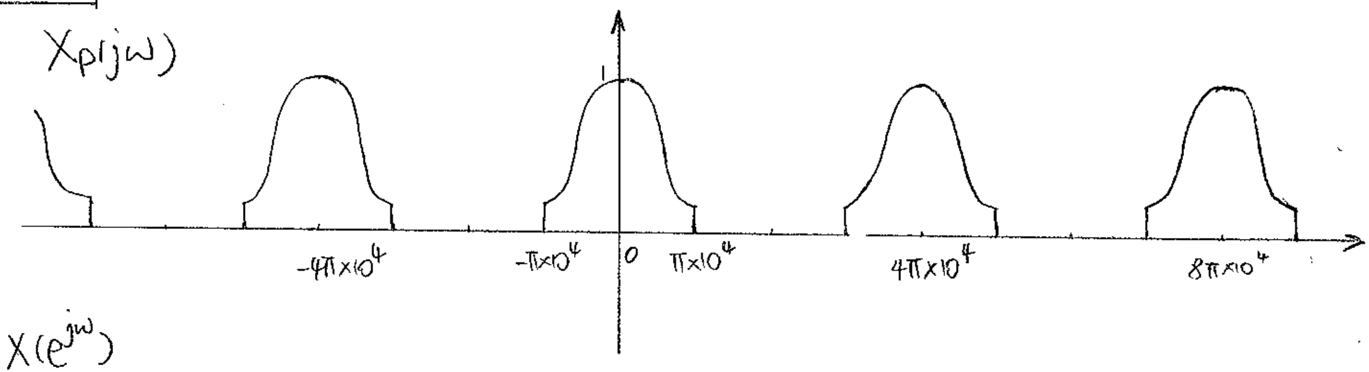
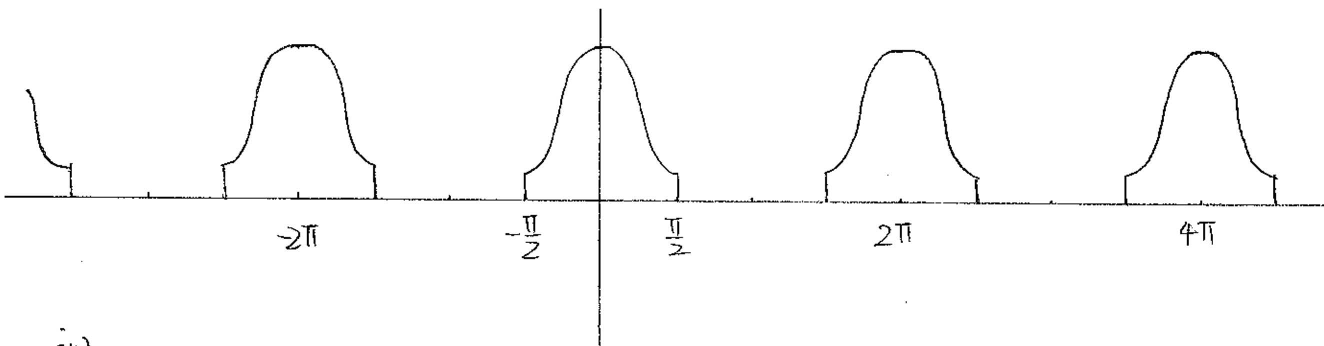
$\boxed{7.29}$

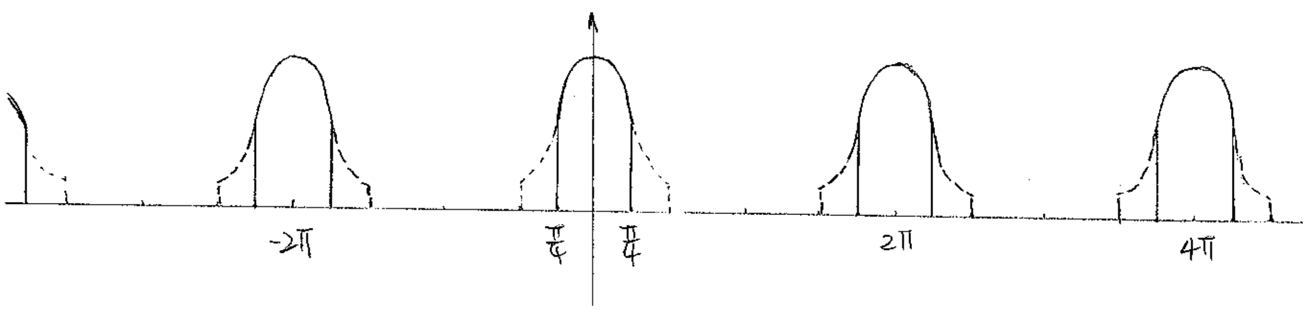
$X_p(j\omega)$



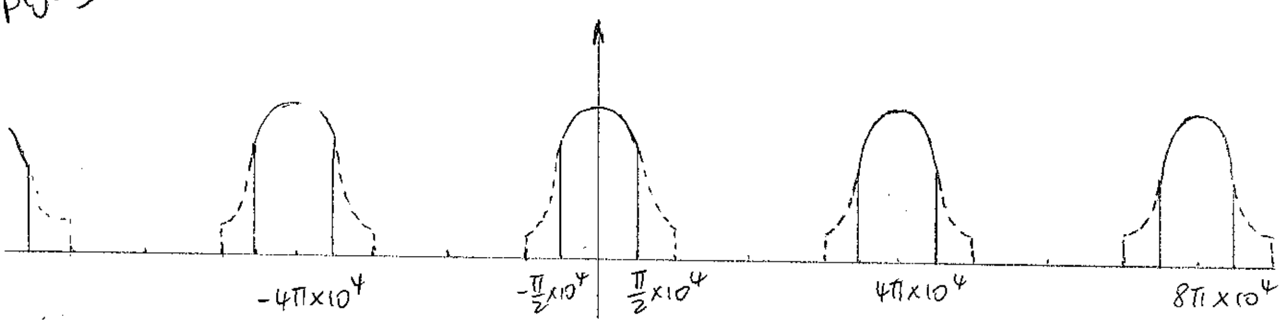
$X(e^{j\omega})$



$Y(e^{j\omega})$



$Y_p(j\omega)$



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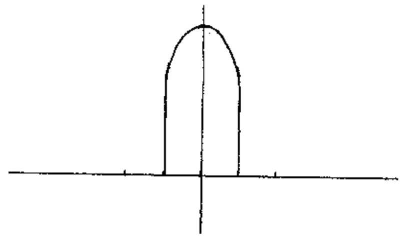
$Y_c(j\omega)$



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$\boxed{7.31}$

$$y[n] = \frac{1}{2} y[n-1] + x[n].$$

$$Y(e^{j\omega}) = \frac{1}{2} e^{-j\omega} Y(e^{j\omega}) + X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2} e^{-j\omega}}.$$

$$X(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c[j(\omega - 2\pi k)/T].$$

$$Y(e^{j\omega}) = \frac{1}{T} \sum Y_c[j(\omega - 2\pi k)/T].$$

$$\therefore \frac{1}{1 - \frac{1}{2} e^{-j\omega}} \sum_{k=-\infty}^{\infty} X_c[j(\omega - 2\pi k)/T] = \sum_{k=-\infty}^{\infty} Y_c[j(\omega - 2\pi k)/T].$$

This requires.

$$X_c(j\omega) \cdot \frac{1}{1 - \frac{1}{2} e^{-j\omega T}} = Y_c(j\omega).$$

$$H_c(j\omega) = \frac{1}{1 - \frac{1}{2} e^{-j\omega T}}.$$

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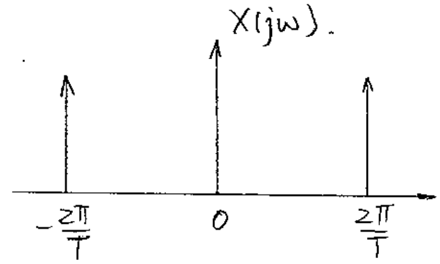
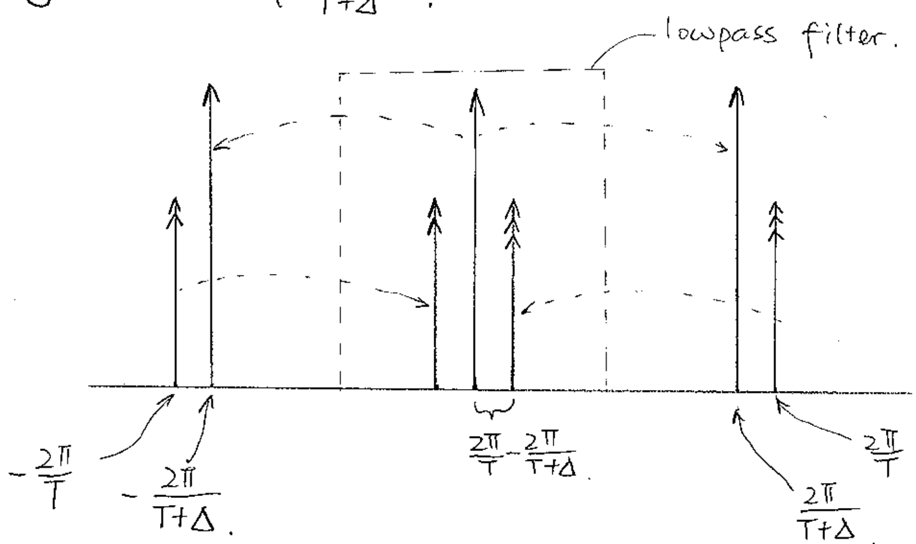
$\boxed{7.38}$

$$x(t) = A + B\cos[(2\pi/T)t + \theta]$$
$$= A + B\,\frac{e^{j[(2\pi/T)t+\theta]} + e^{-j[(2\pi/T)t+\theta]}}{2}$$
$$= A + \frac{Be^{j\theta}}{2}\cdot e^{j(2\pi/T)t} + \frac{Be^{-j\theta}}{2}\, e^{-j(2\pi/T)t}$$

$$X(j\omega) = A\,\delta(\omega) + \frac{Be^{j\theta}}{2}\,\delta(\omega - 2\pi/T) + \frac{Be^{-j\theta}}{2}\,\delta(\omega + \tfrac{2\pi}{T}).$$



The impulse train $p(t) = \sum_{-\infty}^{\infty} \delta[t - n(T+\Delta)]$ shifts the spectrum by multiples of $\frac{2\pi}{T+\Delta}$.



When Δ is very small, the shifted spectrum looks like the one above. The output spectrum is,

$$Y(j\omega) = A\delta(\omega) + \frac{Be^{j\theta}}{2}\,\delta[\omega - (2\pi/T - 2\pi/(T+\Delta))] + \frac{Be^{-j\theta}}{2}\,\delta[\omega + (2\pi/T - 2\pi/(T+\Delta))]$$

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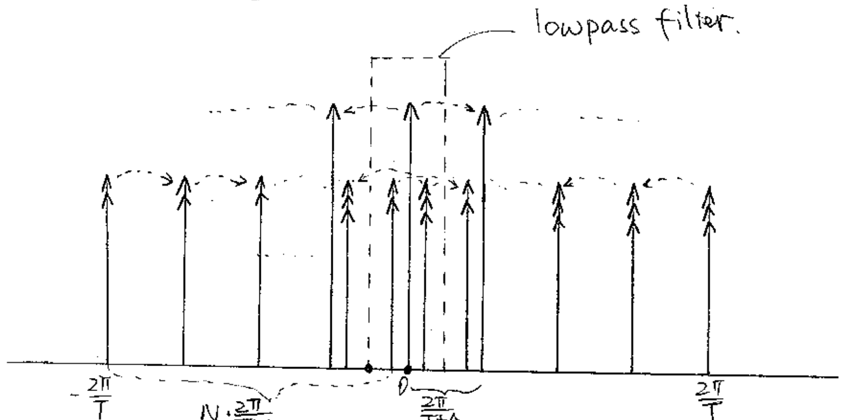
$$y(t) = A + B\cos\left[\left(\frac{2\pi}{T} - \frac{2\pi}{T+\Delta}\right)t + \theta\right].$$

$$\therefore\ a = \frac{\frac{2\pi}{T} - \frac{2\pi}{T+\Delta}}{\frac{2\pi}{T}} = \frac{\Delta}{T+\Delta}.$$

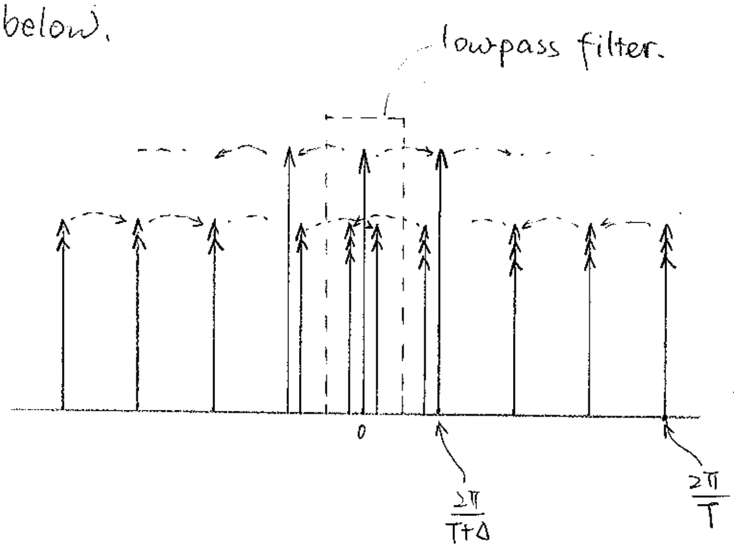
When Δ is large, the situation is more complex,



As could be seen, there will also be side-band spectrum falling within the low-pass filter. But this would require that

$$\frac{2\pi}{T} - \frac{1}{2}\cdot\frac{2\pi}{T+\Delta} < N\cdot\frac{2\pi}{T+\Delta} < \frac{2\pi}{T}, \quad N = 1, 2, 3, \ldots$$

Otherwise the two side bands would exchange their position, and one would not be able to get $y(t) = x(at)$. Such a case is shown below.



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So the range of proper Δ is.

$$(N-1)T < \Delta < 2NT. \quad N = 1, 2, 3, \cdots$$