ECE 301

Division 1, Spring 2007

Instructor: Mimi Boutin

Final Examination

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
3. At the end of this document is a 5 page table of formulas and 4 pages of scratch paper. You may detach these **once the exam begins**. Each formula is labeled with a number. To save time, you may use these numbers to specify which formula you are using. If you use a formula which is *not* contained in this table, you must explain why it is true in order to get full credit.
4. This is a closed book exam. The only personal items allowed are pens/pencils, erasers and something to drink. Anything else is strictly forbidden.
5. You must keep your eyes on your exam at all times. Looking around is strictly forbidden.

Name: ______________________________

Email: ______________________________

Signature: ______________________________

Itemized Scores

Problem 1:	Problem 6:
Problem 2:	Problem 7:
Problem 3:	Problem 8:
Problem 4:	Problem 9:
Problem 5:	
Total:	

(10 pts) **1.** State the sampling theorem. (You may use your own words but be precise!)

2.
(5 pts) **a)** Is the DT signal $x[n] = \cos(6n + \frac{\pi}{6})$ periodic? (Answer yes/no and justify your answer.)

(10 pts) **b)** Is the CT signal $x(t) = e^{2\pi t}$ periodic? (Answer yes/no and justify your answer.)

3.

(5 pts) **a) Without using the table**, find the unit impulse response of the DT system defined by

$$y[n] = x[n-1] - x[n] + 3x[n+2]$$

(10 pts) **b) Without using the table**, find the unit impulse response of the CT system consisting of the cascade of S_1 and S_2, (i.e. S_1 followed by S_2), where S_1 is defined by

$$y(t) = x(t+3),$$

and S_2 is defined by

$$y(t) = x(4t).$$

(Justify your answer.)

(15 pts) **c) Using the table of Laplace transforms and properties**, find the unit impulse response of the CT causal system defined by

$$y'(t) + y(t) = 4x(t)$$

(Justify your answer.)

(15 pts) **4.** An LTI system has unit impulse response $h(t) = u(t+3)$. Compute the system's response to the input $x(t) = e^{-3t}u(t+1)$. (Justify your answer carefully!)

(15 pts) **5.** Compute the Fourier series coefficients of the DT signal $x[n] = (-j)^{n+1}$.

(20 pts) **6.** A DT signal x[n] has z-transform

$$X(z) = \frac{z}{1+z^5}, \qquad \text{ROC } |z| > 1.$$

Use the definition of the z-transform to find x[n].

(15 pts) **7.** The Laplace transform of the unit impulse response of an LTI system is

$$H(s) = \frac{1}{s+2}, \ \mathrm{Re}(s) > -2.$$

Determine the response $y(t)$ of the system when the input is

$$x(t) = \begin{cases} e^{-3t} & t > 0 \\ 0 & t = 0 \\ -e^{3t} & t < 0 \end{cases} .$$

(10 pts) **8.** A signal $x(t)$ has Nyquist rate equal to 14. This signal is modulated by a carrier $c(t) = \cos(10t)$ to get $y(t)$. Draw a diagram representing a system which would demodulate $x(t)$.

(20 pts) **9.** A DT signal $x[n]$ has a Fourier transform $\mathcal{X}(\omega)$ that is zero for $\frac{\pi}{4} \leq |\omega| \leq \pi$. Another signal

$$g[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-1-4k]$$

is generated. Specify the frequency response $H(e^{j\omega})$ of a low-pass filter that produces $x[n]$ when $g[n]$ is the input.

Table

DT Signal Energy and Power

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 \tag{1}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \tag{2}$$

CT Signal Energy and Power

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \tag{3}$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \tag{4}$$

Fourier Series of CT Periodic Signals with period T

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t} \tag{5}$$

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\left(\frac{2\pi}{T}\right)t} dt \tag{6}$$

Fourier Series of DT Periodic Signals with period N

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\left(\frac{2\pi}{N}\right)n} \tag{7}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n} \tag{8}$$

CT Fourier Transform

$$\text{F.T.}: X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \quad (9)$$

$$\text{Inverse F.T.}: x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega \quad (10)$$

Properties of CT Fourier Transform

Let $x(t)$ be a continuous-time signal and denote by $X(\omega)$ its Fourier transform. Let $y(t)$ be another continuous-time signal and denote by $Y(\omega)$ its Fourier transform.

	Signal	*FT*	
Linearity:	$ax(t) + by(t)$	$aX(\omega) + bY(\omega)$	(11)
Time Shifting:	$x(t - t_0)$	$e^{-j\omega t_0}X(\omega)$	(12)
Frequency Shifting:	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$	(13)
Time and Frequency Scaling:	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	(14)
Multiplication:	$x(t)y(t)$	$\frac{1}{2\pi}X(\omega) * Y(\omega)$	(15)
Convolution:	$x(t) * y(t)$	$X(\omega)Y(\omega)$	(16)
Differentiation in Time:	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$	(17)

Some CT Fourier Transform Pairs

$$e^{j\omega_0 t} \xrightarrow{\mathcal{F}} 2\pi\delta(\omega - \omega_0) \quad (18)$$

$$1 \xrightarrow{\mathcal{F}} 2\pi\delta(\omega) \quad (19)$$

$$\frac{\sin Wt}{\pi t} \xrightarrow{\mathcal{F}} u(\omega + W) - u(\omega - W) \quad (20)$$

$$u(t + T_1) - u(t - T_1) \xrightarrow{\mathcal{F}} \frac{2\sin(\omega T_1)}{\omega} \quad (21)$$

$$\delta(t) \xrightarrow{\mathcal{F}} 1 \quad (22)$$

$$e^{-at}u(t), \mathcal{R}e\{a\} > 0 \xrightarrow{\mathcal{F}} \frac{1}{a + j\omega} \quad (23)$$

$$te^{-at}u(t), \mathcal{R}e\{a\} > 0 \xrightarrow{\mathcal{F}} \frac{1}{(a + j\omega)^2} \quad (24)$$

DT Fourier Transform

Let $x[n]$ be a discrete-time signal and denote by $X(\omega)$ its Fourier transform.

$$\text{F.T.:} X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \quad (25)$$

$$\text{Inverse F.T.: } x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega n}d\omega \quad (26)$$

Properties of DT Fourier Transform

Let $x(t)$ be a signal and denote by $X(\omega)$ its Fourier transform. Let $y(t)$ be another signal and denote by $Y(\omega)$ its Fourier transform.

	Signal	*F.T.*	
Linearity:	$ax[n] + by[n]$	$aX(\omega) + bY(\omega)$	(27)
Time Shifting:	$x[n - n_0]$	$e^{-j\omega n_0}X(\omega)$	(28)
Frequency Shifting:	$e^{j\omega_0 n}x[n]$	$X(\omega - \omega_0)$	(29)
Time Reversal:	$x[-n]$	$X(-\omega)$	(30)
Multiplication:	$x[n]y[n]$	$\frac{1}{2\pi}X(\omega) * Y(\omega)$	(31)
Convolution:	$x[n] * y[n]$	$X(\omega)Y(\omega)$	(32)
Differencing in Time:	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(\omega)$	(33)

Some DT Fourier Transform Pairs

$$\sum_{k=0}^{N-1} a_k e^{jk(\frac{2\pi}{N})n} \xrightarrow{\mathcal{F}} 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - \frac{2\pi k}{N}) \quad (34)$$

$$e^{j\omega_0 n} \xrightarrow{\mathcal{F}} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l) \quad (35)$$

$$1 \xrightarrow{\mathcal{F}} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l) \quad (36)$$

$$\frac{\sin Wn}{\pi n}, 0 < W < \pi \xrightarrow{\mathcal{F}} u(\omega + W) - u(\omega - W)X(\omega) = \begin{cases} 1, & 0 \le |\omega| < W \\ 0, & \pi \ge |\omega| > W \end{cases} \quad (37)$$

$$X(\omega)\text{periodic with period } 2\pi$$

$$\delta[n] \xrightarrow{\mathcal{F}} 1 \quad (38)$$

$$u[n] \xrightarrow{\mathcal{F}} \frac{1}{1 - e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k) \quad (39)$$

$$\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{1 - \alpha e^{-j\omega}} \quad (40)$$

$$(n+1)\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{(1 - \alpha e^{-j\omega})^2} \quad (41)$$

Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \tag{42}$$

Properties of Laplace Transform

Let $x(t)$, $x_1(t)$ and $x_2(t)$ be three CT signals and denote by $X(s)$, $X_1(s)$ and $X_2(s)$ their respective Laplace transform. Let R be the ROC of $X(s)$, let R_1 be the ROC of $X_1(z)$ and let R_2 be the ROC of $X_2(s)$.

	Signal	L.T.	ROC	
Linearity:	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$	(43)
Time Shifting:	$x(t - t_0)$	$e^{-st_0}X(s)$	R	(44)
Shifting in s:	$e^{s_0 t}x(t)$	$X(s - s_0)$	$R + s_0$	(45)
Conjugation:	$x^*(t)$	$X^*(s^*)$	R	(46)
Time Scaling:	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	aR	(47)
Convolution:	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$	(48)
Differentiation in Time:	$\frac{d}{dt}x(t)$	$sX(s)$	At least R	(49)
Differentiation in s:	$-tx(t)$	$\frac{dX(s)}{ds}$	R	(50)
Integration :	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \mathcal{R}e\{s\} > 0$	(51)

Some Laplace Transform Pairs

Signal	*LT*	*ROC*	
$\delta(t)$	1	all s	(52)
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\mathcal{R}e\{s\} > -\alpha$	(53)
$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\mathcal{R}e\{s\} < -\alpha$	(54)

z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \quad (55)$$

Properties of z-Transform

Let $x[n]$, $x_1[n]$ and $x_2[n]$ be three DT signals and denote by $X(z)$, $X_1(z)$ and $X_2(z)$ their respective z-transform. Let R be the ROC of $X(z)$, let R_1 be the ROC of $X_1(z)$ and let R_2 be the ROC of $X_2(z)$.

	Signal	z-T.	ROC	
Linearity:	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least $R_1 \cap R_2$	(56)
Time Shifting:	$x[n-n_0]$	$z^{-n_0}X(z)$	R, but perhaps adding/deleting $z=0$	(57)
Time Shifting:	$x[-n]$	$X(z^{-1})$	R^{-1}	(58)
Scaling in z:	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R	(59)
Conjugation:	$x^*(t)$	$X^*(z^*)$	R	(60)
Convolution:	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least $R_1 \cap R_2$	(61)

Some z-Transform Pairs

Signal	LT	ROC	
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$	(62)
$-u(-n-1)$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$	(63)
$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| > \alpha$	(64)
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\|z\| < \alpha$	(65)
$\delta[n]$	1	all z	(66)

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