Instructions:

1. Wait for the “BEGIN” signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.

2. **When the end of the exam is announced, you must stop writing immediately.** Anyone caught writing after the exam is over will get a grade of zero.

3. At the end of this document is a 5 page table of formulas and 4 pages of scratch paper. You may detach these **once the exam begins.** Each formula is labeled with a number. To save time, you may use these numbers to specify which formula you are using. If you use a formula which is not contained in this table, you must explain why it is true in order to get full credit.

4. This is a closed book exam. The only personal items allowed are pens/pencils, erasers and something to drink. Anything else is strictly forbidden.

5. You must keep your eyes on your exam at all times. Looking around is strictly forbidden.

Name: __________________________

Email: __________________________

Signature: ________________________

<table>
<thead>
<tr>
<th>Itemized Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1:</td>
</tr>
<tr>
<td>Problem 2:</td>
</tr>
<tr>
<td>Problem 3:</td>
</tr>
<tr>
<td>Problem 4:</td>
</tr>
<tr>
<td>Problem 5:</td>
</tr>
<tr>
<td>Total:</td>
</tr>
</tbody>
</table>
(10 pts) 1. State the sampling theorem. (You may use your own words but be precise!)
2. (5 pts) a) Is the DT signal $x[n] = \cos(6n + \frac{\pi}{6})$ periodic? (Answer yes/no and justify your answer.)

(10 pts) b) Is the CT signal $x(t) = e^{2\pi t}$ periodic? (Answer yes/no and justify your answer.)
3. (5 pts) **a) Without using the table**, find the unit impulse response of the DT system defined by

\[ y[n] = x[n - 1] - x[n] + 3x[n + 2] \]

(10 pts) **b) Without using the table**, find the unit impulse response of the CT system consisting of the cascade of \( S_1 \) and \( S_2 \), (i.e. \( S_1 \) followed by \( S_2 \)), where \( S_1 \) is defined by

\[ y(t) = x(t + 3), \]

and \( S_2 \) is defined by

\[ y(t) = x(4t). \]

(Justify your answer.)
(15 pts) c) Using the table of Laplace transforms and properties, find the unit impulse response of the CT causal system defined by

\[ y'(t) + y(t) = 4x(t) \]

(Justify your answer.)
(15 pts) 4. An LTI system has unit impulse response $h(t) = u(t+3)$. Compute the system's response to the input $x(t) = e^{-3t}u(t+1)$. (Justify your answer carefully!)
(15 pts) 5. Compute the Fourier series coefficients of the DT signal $x[n] = (-j)^{n+1}$. 
(20 pts) 6. A DT signal $x[n]$ has z-transform

$$X(z) = \frac{z}{1 + z^6}, \quad \text{ROC } |z| > 1.$$ 

Use the definition of the z-transform to find $x[n]$. 
(15 pts) 7. The Laplace transform of the unit impulse response of an LTI system is

\[ H(s) = \frac{1}{s + 2}, \quad \text{Re}(s) > -2. \]

Determine the response \( y(t) \) of the system when the input is

\[ x(t) = \begin{cases} 
  e^{-3t} & t > 0 \\
  0 & t = 0 \\
  -e^{3t} & t < 0 
\end{cases}. \]
(10 pts) **8.** A signal $x(t)$ has Nyquist rate equal to 14. This signal is modulated by a carrier $c(t) = \cos(10t)$ to get $y(t)$. Draw a diagram representing a system which would demodulate $x(t)$. 
(20 pts) 9. A DT signal $x[n]$ has a Fourier transform $X(\omega)$ that is zero for $\frac{\pi}{4} \leq |\omega| \leq \pi$. Another signal

$$g[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n - 1 - 4k]$$

is generated. Specify the frequency response $H(e^{j\omega})$ of a low-pass filter that produces $x[n]$ when $g[n]$ is the input.
Table

DT Signal Energy and Power

\[ E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 \]  \hspace{1cm} (1)

\[ P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \]  \hspace{1cm} (2)

CT Signal Energy and Power

\[ E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \]  \hspace{1cm} (3)

\[ P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \]  \hspace{1cm} (4)

Fourier Series of CT Periodic Signals with period \( T \)

\[ x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \]  \hspace{1cm} (5)

\[ a_k = \frac{1}{T} \int_{0}^{T} x(t)e^{-jk(2\pi/T)t} \, dt \]  \hspace{1cm} (6)

Fourier Series of DT Periodic Signals with period \( N \)

\[ x[n] = \sum_{k=0}^{N-1} a_k e^{jk(2\pi/N)n} \]  \hspace{1cm} (7)

\[ a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n]e^{-jk(2\pi/N)n} \]  \hspace{1cm} (8)
CT Fourier Transform

F.T. : \( X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \) \hspace{1cm} (9)

Inverse F.T.: \( x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \) \hspace{1cm} (10)

Properties of CT Fourier Transform

Let \( x(t) \) be a continuous-time signal and denote by \( X(\omega) \) its Fourier transform. Let \( y(t) \) be another continuous-time signal and denote by \( Y(\omega) \) its Fourier transform.

- **Linearity:**
  \[ ax(t) + by(t) \rightarrow aX(\omega) + bY(\omega) \] \hspace{1cm} (11)

- **Time Shifting:**
  \[ x(t - t_0) e^{-j\omega t_0} \rightarrow X(\omega - \omega_0) \] \hspace{1cm} (12)

- **Frequency Shifting:**
  \[ e^{j\omega_0 t} x(t) \rightarrow X(\omega - \omega_0) \] \hspace{1cm} (13)

- **Time and Frequency Scaling:**
  \[ x(at) \rightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right) \] \hspace{1cm} (14)

- **Multiplication:**
  \[ x(t)y(t) \rightarrow \frac{1}{2\pi} X(\omega) * Y(\omega) \] \hspace{1cm} (15)

- **Convolution:**
  \[ x(t) * y(t) \rightarrow X(\omega)Y(\omega) \] \hspace{1cm} (16)

- **Differentiation in Time:**
  \[ \frac{d}{dt} x(t) \rightarrow j\omega X(\omega) \] \hspace{1cm} (17)

Some CT Fourier Transform Pairs

\[ e^{j\omega_0 t} \xrightarrow{\mathcal{F}} 2\pi \delta(\omega - \omega_0) \] \hspace{1cm} (18)

\[ 1 \xrightarrow{\mathcal{F}} 2\pi \delta(\omega) \] \hspace{1cm} (19)

\[ \frac{\sin W t}{\pi t} \xrightarrow{\mathcal{F}} u(\omega + W) - u(\omega - W) \] \hspace{1cm} (20)

\[ u(t + T_1) - u(t - T_1) \xrightarrow{\mathcal{F}} \frac{2\sin(\omega T_1)}{\omega} \] \hspace{1cm} (21)

\[ \delta(t) \xrightarrow{\mathcal{F}} 1 \] \hspace{1cm} (22)

\[ e^{-at} u(t), \Re\{a\} > 0 \xrightarrow{\mathcal{F}} \frac{1}{a + j\omega} \] \hspace{1cm} (23)

\[ te^{-at} u(t), \Re\{a\} > 0 \xrightarrow{\mathcal{F}} \frac{1}{(a + j\omega)^2} \] \hspace{1cm} (24)
DT Fourier Transform

Let \( x[n] \) be a discrete-time signal and denote by \( X(\omega) \) its Fourier transform.

\[
\text{F.T.: } X(\omega) = \sum_{n=\infty}^{\infty} x[n]e^{-jn\omega} \quad (25)
\]

\[
\text{Inverse F.T.: } x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega)e^{jn\omega}d\omega \quad (26)
\]

Properties of DT Fourier Transform

Let \( x(t) \) be a signal and denote by \( X(\omega) \) its Fourier transform. Let \( y(t) \) be another signal and denote by \( Y(\omega) \) its Fourier transform.

\[
\begin{align*}
\text{Signal} & \quad \text{F.T.} \\
\text{Linearity: } ax[n] + by[n] & \quad aX(\omega) + bY(\omega) \quad (27) \\
\text{Time Shifting: } x[n - n_0] & \quad e^{-jn_0\omega}X(\omega) \quad (28) \\
\text{Frequency Shifting: } e^{j\omega_0 n}x[n] & \quad X(\omega - \omega_0) \quad (29) \\
\text{Time Reversal: } x[-n] & \quad X(-\omega) \quad (30) \\
\text{Multiplication: } x[n]y[n] & \quad \frac{1}{2\pi}X(\omega) * Y(\omega) \quad (31) \\
\text{Convolution: } x[n] * y[n] & \quad X(\omega)Y(\omega) \quad (32) \\
\text{Differencing in Time: } x[n] - x[n-1] & \quad (1 - e^{-j\omega})X(\omega) \quad (33)
\end{align*}
\]

Some DT Fourier Transform Pairs

\[
\begin{align*}
\sum_{k=0}^{N-1} a_ke^{j\omega k}(\frac{\pi}{N})^n & \quad \mathcal{F} \quad 2\pi \sum_{k=\infty}^{\infty} a_k\delta(\omega - \frac{2\pi k}{N}) \quad (34) \\
e^{j\omega_0 n} & \quad \mathcal{F} \quad 2\pi \sum_{l=\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l) \quad (35) \\
1 & \quad \mathcal{F} \quad 2\pi \sum_{l=\infty}^{\infty} \delta(\omega - 2\pi l) \quad (36) \\
\frac{\sin Wn}{\pi n}, 0 < W < \pi & \quad \mathcal{F} \quad u(\omega + W) - u(\omega - W)X(\omega) = \begin{cases} 1, & 0 \leq |\omega| < W \\ 0, & \pi \geq |\omega| > W \end{cases} \quad (37) \\
\delta[n] & \quad \mathcal{F} \quad 1 \quad (38) \\
u[n] & \quad \mathcal{F} \quad \frac{1}{1 - e^{-j\omega}} + \pi \sum_{k=\infty}^{\infty} \delta(\omega - 2\pi k) \quad (39) \\
\alpha^n u[n], |\alpha| < 1 & \quad \mathcal{F} \quad \frac{1}{1 - \alpha e^{-j\omega}} \quad (40) \\
(n + 1)\alpha^n u[n], |\alpha| < 1 & \quad \mathcal{F} \quad \frac{1}{(1 - \alpha e^{-j\omega})^2} \quad (41)
\end{align*}
\]

14
Laplace Transform

\[ X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \]  \hspace{1cm} (42)

Properties of Laplace Transform

Let \( x(t) \), \( x_1(t) \) and \( x_2(t) \) be three CT signals and denote by \( X(s) \), \( X_1(s) \) and \( X_2(s) \) their respective Laplace transform. Let \( R \) be the ROC of \( X(s) \), let \( R_1 \) be the ROC of \( X_1(z) \) and let \( R_2 \) be the ROC of \( X_2(s) \).

<table>
<thead>
<tr>
<th>Signal</th>
<th>L.T.</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity: ( ax_1(t) + bx_2(t) )</td>
<td>( aX_1(s) + bX_2(s) )</td>
<td>At least ( R_1 \cap R_2 )  \hspace{1cm} (43)</td>
</tr>
<tr>
<td>Time Shifting: ( x(t - t_0) )</td>
<td>( e^{-st_0}X(s) )</td>
<td>( R )                       \hspace{1cm} (44)</td>
</tr>
<tr>
<td>Shifting in ( s ): ( e^{st}x(t) )</td>
<td>( X(s - s_0) )</td>
<td>( R + s_0 )                 \hspace{1cm} (45)</td>
</tr>
<tr>
<td>Conjugation: ( x^*(t) )</td>
<td>( X^<em>(s^</em>) )</td>
<td>( R )                       \hspace{1cm} (46)</td>
</tr>
<tr>
<td>Time Scaling: ( x(at) )</td>
<td>( \frac{1}{</td>
<td>a</td>
</tr>
<tr>
<td>Convolution: ( x_1(t) \ast x_2(t) )</td>
<td>( X_1(s)X_2(s) )</td>
<td>At least ( R_1 \cap R_2 )  \hspace{1cm} (48)</td>
</tr>
<tr>
<td>Differentiation in Time: ( \frac{d}{dt}x(t) )</td>
<td>( sX(s) )</td>
<td>At least ( R )              \hspace{1cm} (49)</td>
</tr>
<tr>
<td>Differentiation in ( s ): ( -tx(t) )</td>
<td>( \frac{dX(s)}{ds} )</td>
<td>( R )                       \hspace{1cm} (50)</td>
</tr>
<tr>
<td>Integration: ( \int_{-\infty}^{t} x(\tau)d\tau )</td>
<td>( \frac{1}{s}X(s) )</td>
<td>At least ( R \cap \Re{s} &gt; 0 )  \hspace{1cm} (51)</td>
</tr>
</tbody>
</table>

Some Laplace Transform Pairs

<table>
<thead>
<tr>
<th>Signal ( \delta(t) )</th>
<th>( 1 )</th>
<th>( \text{all } s )</th>
<th>(52)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^{-\alpha t}u(t) )</td>
<td>( \frac{1}{s + \alpha} )</td>
<td>( \Re{s} &gt; -\alpha )</td>
<td>(53)</td>
</tr>
<tr>
<td>( -e^{-\alpha t}u(-t) )</td>
<td>( \frac{1}{s + \alpha} )</td>
<td>( \Re{s} &lt; -\alpha )</td>
<td>(54)</td>
</tr>
</tbody>
</table>
z-Transform

\[ X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \quad (55) \]

Properties of z-Transform

Let \( x[n] \), \( x_1[n] \) and \( x_2[n] \) be three DT signals and denote by \( X(z) \), \( X_1(z) \) and \( X_2(z) \) their respective z-transform. Let \( R \) be the ROC of \( X(z) \), let \( R_1 \) be the ROC of \( X_1(z) \) and let \( R_2 \) be the ROC of \( X_2(z) \).

<table>
<thead>
<tr>
<th>Signal</th>
<th>z-T.</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearity: ( ax_1[n] + bx_2[n] )</td>
<td>( aX_1(z) + bX_2(z) )</td>
<td>At least ( R_1 \cap R_2 ) (56)</td>
</tr>
<tr>
<td>Time Shifting: ( x[n - n_0] )</td>
<td>( z^{-n_0}X(z) )</td>
<td>( R ), but perhaps adding/deleting ( z = 0 ) (57)</td>
</tr>
<tr>
<td>Time Shifting: ( x[-n] )</td>
<td>( X(z^{-1}) )</td>
<td>( R^{-1} ) (58)</td>
</tr>
<tr>
<td>Scaling in z: ( e^{j\omega_0 n}x[n] )</td>
<td>( X(e^{-j\omega_0 z}) )</td>
<td>( R ) (59)</td>
</tr>
<tr>
<td>Conjugation: ( x^*(t) )</td>
<td>( X^<em>(z^</em>) )</td>
<td>( R ) (60)</td>
</tr>
<tr>
<td>Convolution: ( x_1[n] \ast x_2[n] )</td>
<td>( X_1(z)X_2(z) )</td>
<td>At least ( R_1 \cap R_2 ) (61)</td>
</tr>
</tbody>
</table>

Some z-Transform Pairs

<table>
<thead>
<tr>
<th>Signal</th>
<th>LT</th>
<th>ROC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u[n] )</td>
<td>( \frac{1}{1 - z^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>( -u[-n - 1] )</td>
<td>( \frac{1}{1 - z^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>( \alpha^n u[n] )</td>
<td>( \frac{1}{1 - \alpha z^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>( -\alpha^n u[-n - 1] )</td>
<td>( \frac{1}{1 - \alpha z^{-1}} )</td>
<td>(</td>
</tr>
<tr>
<td>( \delta[n] )</td>
<td>( 1 )</td>
<td>all ( z ) (66)</td>
</tr>
</tbody>
</table>
-SCRATCH -
(will not be graded)
-SCRATCH -
(will not be graded)
-SCRATCH -
(will not be graded)