1. Let $A \subset \mathbb{R}$ with $\emptyset \neq A$. Set $\alpha = \sup A$ and suppose $\alpha < \infty$. Also suppose there exists a $\delta > 0$ such that for all distinct a and b in A we have $|a - b| \geq \delta$. Show $\alpha \in A$.

2. Let $A_j \subset \mathbb{R}$ and $\alpha_j = \sup A_j$. Show $\sup(\cup A_j) = \sup\{\alpha_j\}$.

3. Let $a_j \in \mathbb{R}, A_N = \{a_N, a_{N+1}, ...\}$.

   (a) $\inf A_N \leq \inf A_{N+1}$, and $\sup A_N \geq \sup A_{N+1}$.

   (b) For all $M, N \in \mathbb{N}, \inf A_N \leq \sup A_M$. Conclude

   $$\sup_N \inf A_N \leq \inf_N \sup A_N.$$

4. Countable or uncountable (with proof)?

   (a) $\oplus_{\mathbb{N}}\mathbb{Q} = \{(q_1, q_2 ...) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times ... :$ only finitely many q_i are non-zero.$\}$.

   (b) $\Pi_{\mathbb{N}}\mathbb{Q} = \{(q_1, q_2 ...) \in \mathbb{Q} \times \mathbb{Q} \times \mathbb{Q} \times ...\}$