

ECE 495N EXAM I**Friday, Oct.2, 2009**NAME : SOLUTION

PUID # : _____

CLOSED BOOK**Useful relations**

$$I = \frac{2q}{\hbar} \int_{-\infty}^{+\infty} dE D(E-U) \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} [f_1(E) - f_2(E)] \quad (1)$$

$$\text{with } N = 2(\text{for spin}) \int_{-\infty}^{+\infty} dE D(E-U) \frac{\gamma_1 f_1(E) + \gamma_2 f_2(E)}{\gamma_1 + \gamma_2} \quad (2)$$

$$\text{and } U = U_L + U_0(N - N_0) \quad (3)$$

$$\text{Fermi functions : } f_1(E) = \frac{1}{e^{(E-\mu_1)/kT} + 1} \text{ and } f_2(E) = \frac{1}{e^{(E-\mu_2)/kT} + 1} \quad (4)$$

$$\text{Law of equilibrium : } P_\alpha = \frac{1}{Z} \exp(-(E_\alpha - \mu N_\alpha)/kT) \quad (5)$$

Please show all work and write your answers clearly.**This exam should have seven pages.**

Problem 1 [p. 2, 3] 8 points

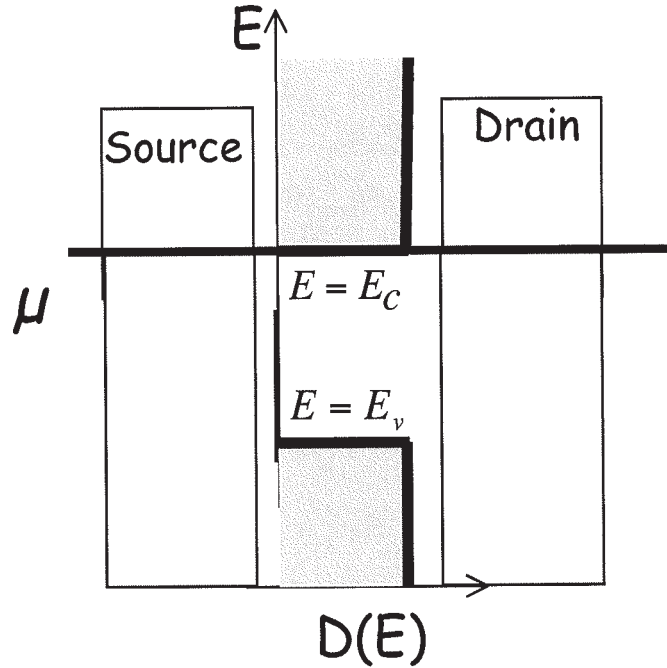
Problem 2 [p. 4, 5] 8 points

Problem 3 [p. 6, 7] 9 points

Total 25 points

Problem 1: Assume $U_0 = 0$ and the Laplace potential U_L to be a fraction α of the drain potential V_D (the source potential is assumed zero):

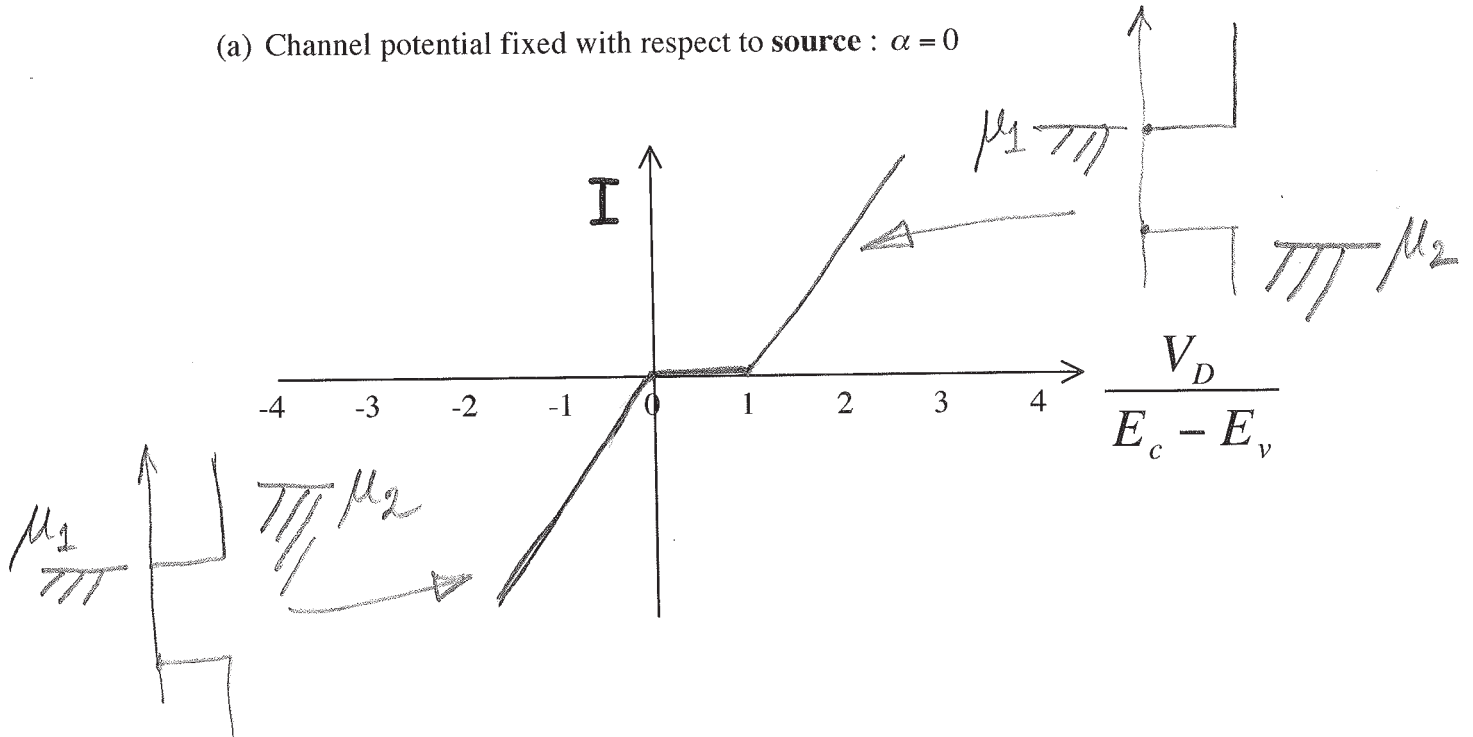
$$U_L = -q \alpha V_D, \alpha \text{ being a constant between 0 and 1.}$$



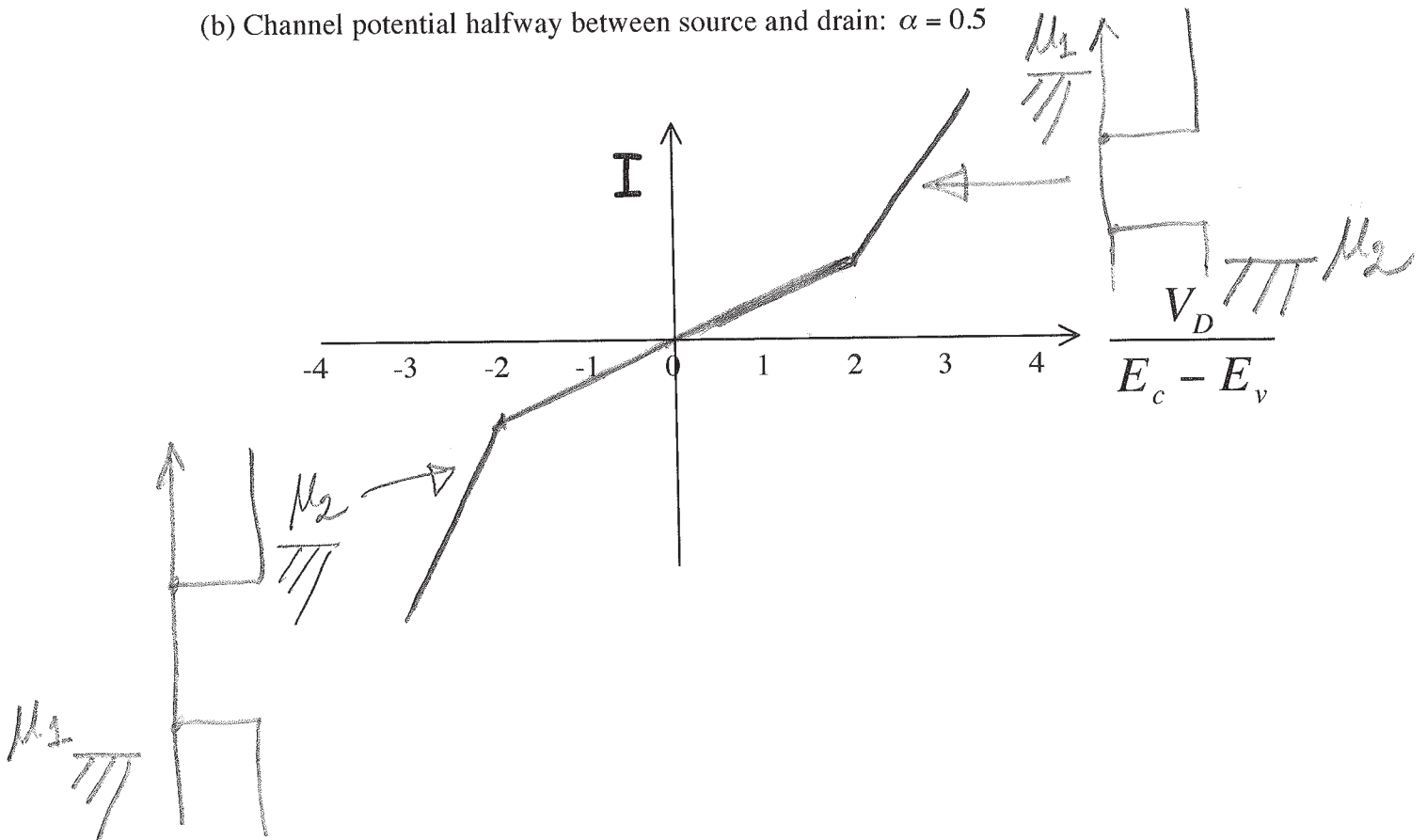
A channel has a density of states as shown, namely a constant non-zero value for $E \geq E_C$ *as well as* for $E < E_v$, with a zero density of states in between. Assume that the equilibrium electrochemical potential μ is located *exactly* at $E = E_C$ as shown.

Sketch the current versus drain voltage assuming that the electrostatic potential of the channel (a) remains fixed with respect to the **source** ($\alpha = 0$) and (b) assumes a value halfway between the source and drain potentials ($\alpha = 0.5$),
Explain your reasoning clearly.

(a) Channel potential fixed with respect to **source** : $\alpha = 0$



(b) Channel potential halfway between source and drain: $\alpha = 0.5$



Problem 2: We have seen in class that free electrons in the absence of any external potential are described by (in three dimensions)

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi \quad (1)$$

whose solutions can be written in the form (ψ_0 being a constant)

$$\psi(x,t) = \psi_0 e^{i\vec{k}\cdot\vec{r}} e^{-iEt/\hbar} \quad \text{where } \vec{k}\cdot\vec{r} = k_x x + k_y y + k_z z \quad (2)$$

with E and k related by the dispersion relation:

$$E = \hbar^2(k_x^2 + k_y^2 + k_z^2)/2m \quad (3)$$

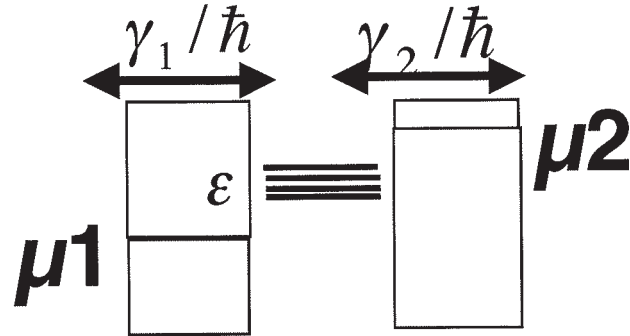
(a) Can you suggest a suitable differential equation to replace (1) if you wanted the dispersion relation to look like $E^2 = m^2 c^4 + \hbar^2 c^2 (k_x^2 + k_y^2 + k_z^2)$ (3') instead of (3) ?

(b) If a system of electrons with a dispersion relation given by (3') were confined in a cubic box of side L, what will be the lowest energy level ?

$$a) \quad \hbar^2 \frac{\partial^2 \psi}{\partial t^2} = \hbar^2 c^2 \nabla^2 \psi - m^2 c^4 \psi$$

$$b) \quad E^2 = m^2 c^4 + \hbar^2 c^2 (k_x^2 + k_y^2 + k_z^2) \\ = m^2 c^4 + 3 \hbar^2 c^2 \left(\frac{\pi}{L} \right)^2$$

$$E = \sqrt{m^2 c^4 + 3 \hbar^2 c^2 \left(\pi/L \right)^2}$$

Problem 3:

A box has four degenerate energy levels all having energy ϵ . We know that for non-interacting electrons the maximum current under bias is $I = \frac{q}{\hbar} \frac{4\gamma_1\gamma_2}{\gamma_1 + \gamma_2}$

regardless of which direction the voltage is applied.

Now assume that the electron-electron interaction energy is so high that no more than one electron can be inside the box at the same time. The maximum current will now be different. Find the appropriate expression for the maximum current

if (a) $\mu_2 > \mu_1$ as shown

if (b) the bias is reversed so that $\mu_1 > \mu_2$

Problem 3:

For short answer: when $\mu_2 > \mu_1$ replace the γ_2 by $4\gamma_2$ then the current will be:

$$I = \frac{q}{\hbar} \frac{\gamma_1 4\gamma_2}{\gamma_1 + 4\gamma_2}$$

when $\mu_1 > \mu_2$ replace the γ_1 by $4\gamma_1$ then the current will be:

$$I = \frac{q}{\hbar} \frac{4\gamma_1 \gamma_2}{4\gamma_1 + \gamma_2}$$

ed
Detail calculation:

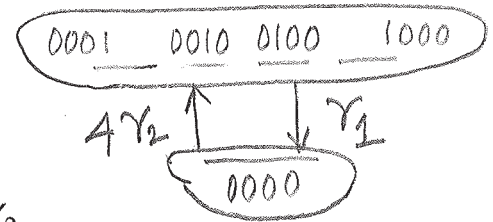
1. When $\mu_2 > \mu_1$:

$$f_1 = 0, f_2 = 1$$

$$4P_{0000}\gamma_2 = (P_{0001} + P_{0010} + P_{0100} + P_{100})\gamma_1 = P_1\gamma_1$$

$$\Rightarrow \frac{P_0}{P_1} = \frac{\gamma_1}{4\gamma_2} \Rightarrow \frac{P_0 + P_1}{P_1} = \frac{\gamma_1 + 4\gamma_2}{4\gamma_2} \Rightarrow P_1 = \frac{4\gamma_2}{4\gamma_2 + \gamma_1}$$

$$\Rightarrow \text{The current } I = \frac{q}{\hbar} P_1 \gamma_1 = \frac{q}{\hbar} \frac{4\gamma_2 \gamma_1}{4\gamma_2 + \gamma_1}$$



2. When $\mu_1 > \mu_2$:

$$f_1 = 1, f_2 = 0$$

$$4P_{0000}\gamma_1 = (P_{0001} + P_{0010} + P_{0100} + P_{100})\gamma_2 = P_1\gamma_2$$

$$\Rightarrow \frac{P_0}{P_1} = \frac{\gamma_2}{4\gamma_1} \Rightarrow \frac{P_0 + P_1}{P_1} = \frac{\gamma_2 + 4\gamma_1}{4\gamma_1} \Rightarrow P_1 = \frac{4\gamma_1}{4\gamma_1 + \gamma_2}$$

$$\Rightarrow \text{The current } I = \frac{q}{\hbar} P_1 \gamma_2 = \frac{q}{\hbar} \frac{4\gamma_1 \gamma_2}{4\gamma_1 + \gamma_2}$$

