

3pts clarity - flow answer.
2pts answer. numeric.

Student Name: _____

Grader Name: _____

+

Instructions: This is an open notes, open book, collaborative quiz. No Internet allowed.

Question: Let A be the event: "it is sunny today." The B be the event: "Joe ate cereal for breakfast today." Assume that A and B are independent events, and that $P(A)=0.5$ and $P(B)=0.3$. What is the probability that only one of these events (either A or B but not both) occurs? Carefully derive the answer, justifying every step along the way.

7

$$P(\text{either A or B but not both}) = P((A \cap B^c) \cup (B \cap A^c))$$

$$= P(A \cap B^c) + P(B \cap A^c), \quad \text{by axiom III since } (A \cap B^c) \cap (B \cap A^c) = \emptyset$$

but A and B^c are independent since

$$P(\Omega | B) = 1 \quad \text{by axiom II}$$

$$\Rightarrow P(A \cup A^c | B) = 1$$

6

$$\Rightarrow P(A|B) + P(A^c|B) = 1 \quad \text{since } A \cap A^c = \emptyset$$

since A and B are independent

$$\Rightarrow P(A) + P(A^c|B) = 1$$

$$\Rightarrow P(A^c|B) = 1 - P(A) = P(A^c)$$

and similarly B and A^c are independent

$$\text{Therefore } P(A \cap B^c) = P(A)P(B^c)$$

$$= P(A)(1 - P(B)) \quad (1)$$

$$= 0.5(0.7)$$

$$= 0.35$$

$$\text{and } P(B \cap A^c) = P(B)P(A^c)$$

$$= P(B)(1 - P(A)) \quad (2)$$

$$= 0.3(0.5)$$

$$= 0.15$$

$$\Rightarrow P(\text{either A or B but not both}) = \cancel{0.15} + 0.35 = 0.5$$

Score: / 20