

10/8 Example

Show that the FT of $\cos 2\pi t$ is

$$\pi \delta(\omega + 2\pi) + \pi \delta(\omega - 2\pi)$$

We show that

I.F.T. of $\pi \delta(\omega + 2\pi) + \pi \delta(\omega - 2\pi)$ is
 $\cos 2\pi t$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} (\pi \delta(\omega + 2\pi) + \pi \delta(\omega - 2\pi)) e^{j\omega t} d\omega =$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \delta(\omega + 2\pi) e^{j\omega t} d\omega + \frac{1}{2} \int_{-\infty}^{\infty} \delta(\omega - 2\pi) e^{j\omega t} d\omega$$

$$= \frac{1}{2} e^{-j2\pi t} + \frac{1}{2} e^{j2\pi t} = \cos(2\pi t)$$

Example

Easy question

The input of an LTI system is $x(t) = 1$
 The unit impulse response of the system is

$$h(t) = \delta(t-3)$$

What is the F.T. of the response $y(t)$ of the system?

$$Y(\omega) = \mathcal{F}\{x(t) * h(t)\}$$

$$= \mathcal{F}\{x(t)\} \mathcal{F}\{h(t)\}$$

$$= \mathcal{F}\{1\} \mathcal{F}\{\delta(t-3)\}$$

$$\downarrow \text{you can do this since } \mathcal{F}^{-1}\{\delta(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi d\omega e^{j\omega t} dt$$

$$= e^0 = 1$$

$$\therefore \mathcal{F}\{1\} = 2\pi \delta(\omega)$$

$$= 2\pi \delta(\omega) e^{-3j\omega} \mathcal{F}\{\delta(t)\}$$

$$= \{2\pi \delta(\omega)\} e^{-3j\omega} \cdot 1$$

$$= 2\pi \delta(\omega) e^0$$

$$= 2\pi \delta(\omega) \quad \text{final solution}$$

* Exercise: compute the F.T. of $u(t-3)$

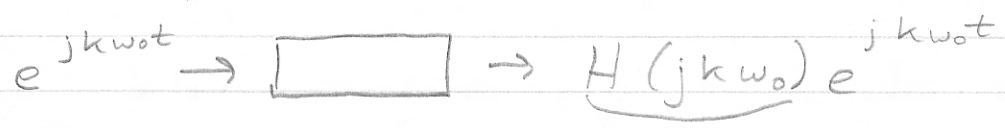
Frequency Response

$H(j\omega)$ in CT
 $H(e^{j\omega})$ in DT



$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$$

when $s = j\omega$
 $H(j\omega)$

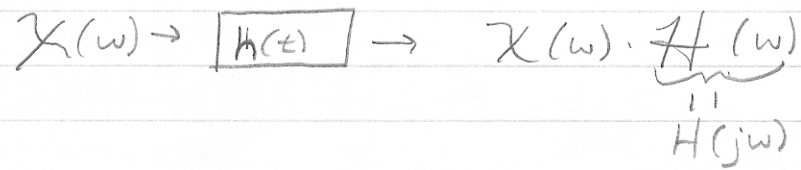


this shows us what the system does to frequency components of the signal

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

F.T. of $h(t)$
 $H(\omega)$





$$X(\omega) \rightarrow \boxed{h(t)} \rightarrow X(\omega) H(j\omega)$$

Example Question

What is the F.T. of the LTI system defined by

$$y(t) = \frac{d}{dt} x(t)$$

$$\mathcal{F}(y(t)) = \mathcal{F}\left(\frac{d}{dt} x(t)\right)$$

$$Y(\omega) = j\omega X(\omega) \quad \text{by the diff. property of FTs}$$

we know that:

note
s is specified
as $s = j\omega$

$$Y(\omega) = H(j\omega) X(\omega)$$

So $H(j\omega) = j\omega$ is the f response to the system

Q2 what is the F.T. of $h(t)$?

$$\mathcal{F}(h(t)) = j\omega$$

$$\mathcal{F}(h(t)) = j\omega$$

$$\boxed{\cancel{H(j\omega) = j \cdot j\omega = -\omega}}$$

this is not the F.T.

Systems characterized by linear, constant coefficient differential equations

what is this?

$$\hookrightarrow \sum_{k=0}^N a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^m b_k \frac{d^k}{dt^k} x(t)$$

Example Question

What is the f response of this system?

FT on left = FT on right

$$\mathcal{F} \left(\sum_{k=0}^N a_k \frac{d^k}{dt^k} y(t) \right) = \mathcal{F} \left(\sum_{k=0}^m b_k \frac{d^k}{dt^k} x(t) \right)$$

$$\Leftrightarrow \sum_{k=0}^N a_k \mathcal{F} \left(\frac{d^k}{dt^k} y(t) \right) = \sum_{k=0}^m b_k \mathcal{F} \left(\frac{d^k}{dt^k} x(t) \right)$$

apply diff property k times

$$\Leftrightarrow \sum_{k=0}^N a_k (j\omega)^k Y(\omega) = \sum_{k=0}^m b_k (j\omega)^k X(\omega)$$

$$\Leftrightarrow Y(\omega) = \frac{\sum_{k=0}^m b_k (j\omega)^k}{\sum_{k=0}^N a_k (j\omega)^k} X(\omega)$$

$H(j\omega) \equiv f$ response

this is true because

$$Y(\omega) = H(j\omega) X(\omega)$$

cont'd

b) find $h(t)$

$$\text{since } H(j\omega) = \mathcal{F}(h(t)) = \mathcal{F}(h(t))$$

$$\Rightarrow h(t) = \mathcal{F}^{-1}(H(j\omega))$$

* exercise:

$$\frac{d}{dt} y(t) + 4y(t) = x(t)$$

a) what is the f response of system?

b) what is the unit impulse response $h(t)$ of the system?

multiplication property

$$\mathcal{F}(X_1(t) \cdot X_2(t)) = \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$$

this is the dual of the convolution property

Note on HW6:

Do all book problems and
hand in half by wed 15th by 5:30
or do all on Rhea

Chapter 5 DT Fourier Transforms

$$X[n] \xrightarrow{\mathcal{F}} X(\omega) \leftarrow \begin{array}{l} \text{this is continuous on } \omega \\ \text{and periodic} \end{array}$$

$$\xleftarrow{\mathcal{F}^{-1}}$$

if the signal is periodic then a Fourier Series will suffice

if the signal is aperiodic then the Fourier Transform must be applied
but it may also be applied to periodic signals.

formulas

$$X(\omega) = \mathcal{F}(X[n]) = \sum_{n=-\infty}^{\infty} X[n] e^{-j\omega n}$$

$$X[n] = \mathcal{F}^{-1}(X(\omega)) = \frac{1}{2\pi} \int_0^{2\pi} X(\omega) e^{j\omega n} d\omega$$