$\boxed{3.22}$

a) $a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt$

$= \frac{1}{2}\int_{-1}^{1} t e^{-jk\omega_0 t} dt$

$= \frac{1}{2}\left\{ \frac{1}{-jk\omega_0} e^{-jk\omega_0 t} \cdot t \Big|_{-1}^{1} - \int_{-1}^{1} e^{-jk\omega_0 t} \cdot dt \right\}$

$= \frac{1}{2}\left\{ \frac{1}{-jk\omega_0}\left[e^{-jk\omega_0} + e^{jk\omega_0 t}\right] - \frac{1}{-jk\omega_0} e^{-jk\omega_0 t}\Big|_{-1}^{1} \right\}$

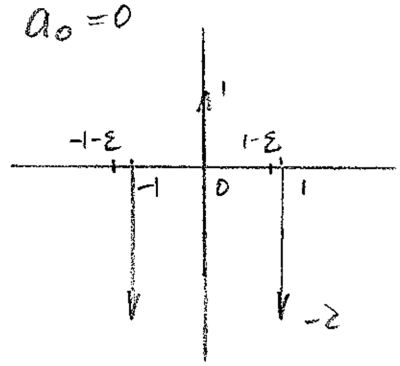
$= \frac{1}{-2jk\omega_0} \cdot \left[2\cos k\omega_0 + 2j\sin k\omega_0 \right]$

$= \frac{e^{j(k+\frac{1}{2})\pi}}{k\pi}$ ⊛

⊛ Note $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$

$a_0 = 0$

d)



$a_k = \frac{1}{2}\int_{-1-\varepsilon}^{1-\varepsilon} \left[\delta(t) - 2\delta(t+1) \right] e^{-jk\omega_0 t} dt$

$= \frac{1}{2}\left[e^{-jk\omega_0 \cdot 0} - 2 \cdot e^{-jk\omega_0 (-1)} \right]$

$= \frac{1}{2}(1 - 2e^{jk\pi})$

$= \frac{1}{2}[1 - (-1)^k]$

$\boxed{3.23}$

b) It is known that $C_k = \dfrac{\sin k\omega_0 T_1}{k\pi}$ is the Fourier Series coefficients of the signal.

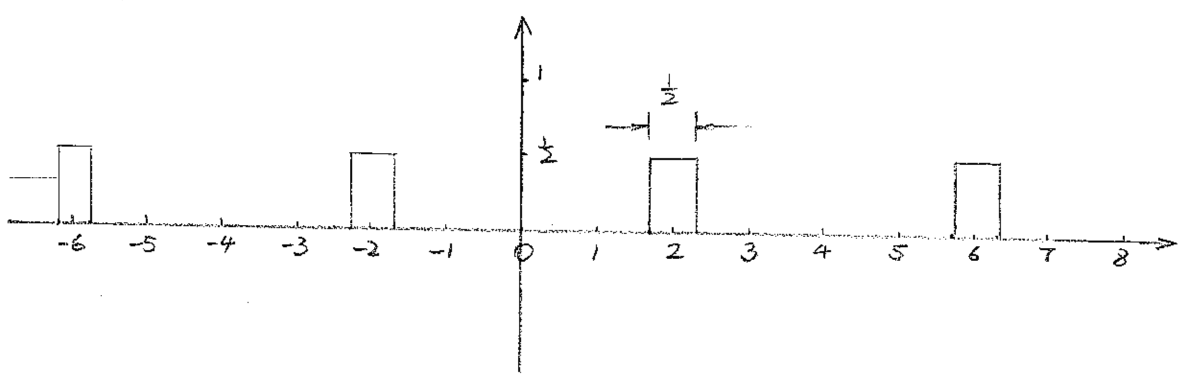
$$x_0(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T}{2} \end{cases} \quad \text{and } x_0(t+T) = x(t).$$

Now that $a_k = (-1)^k \dfrac{\sin k\pi/8}{2k\pi}$

$$= \frac{1}{2} \cdot e^{jk\frac{2\pi}{4}\cdot 2} \cdot \frac{\sin k \cdot \frac{2\pi}{4} \cdot \frac{1}{4}}{k\pi}.$$

we know from the time shifting property of Fourier Series, that a_k must be the Fourier Series coefficients of the signal.

$$x(t) = \begin{cases} \frac{1}{2}, & \frac{7}{4} < t < \frac{9}{4} \\ 0, & -\frac{7}{4} < t < \frac{7}{4} \end{cases} \quad \text{and } x(t+4) = x(t).$$



d)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} \left[e^{j2n\omega_0 t} + 2e^{j(2n+1)\omega_0 t} \right]$$

$$= \sum_{n=-\infty}^{\infty} \left[e^{j\cdot 2n\cdot \frac{2\pi}{4} t} + 2e^{j(2n+1)\frac{2\pi}{4}\cdot t} \right]$$

$$= \sum_{n=-\infty}^{\infty} \left[e^{jn\pi t} + 2e^{j\cdot\frac{\pi}{2} t} \cdot e^{jn\pi t} \right]$$

$$= (1 + 2e^{j\frac{\pi}{2} t}) \sum_{n=-\infty}^{\infty} e^{jn\pi t}$$

$$= (1 + 2e^{j\frac{\pi}{2} t}) \sum_{k=-\infty}^{\infty} \delta(t-2k).$$

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Homework # 5 Solution

3.46

a) $x(t)y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \cdot \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$

$$= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_n b_m e^{j(n+m)\omega_0 t}$$

Let $m+n=k$,

$$x(t)y(t) = \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_n b_{k-n} e^{jk\omega_0 t}$$

$$z(t) = x(t)y(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$\therefore\ c_k = \sum_{n=-\infty}^{\infty} a_n b_{k-n}$$

c_k could be seen as a convolution of a_k and b_k.

b) The period of the signal is 3. $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$

Let $x_1(t) = p(t) q_1(t)$,

where $p(t) = \cos 20\pi t$, $q(t) = \begin{cases} 1, & |t|<1 \\ 0, & 1<|t|<1.5 \end{cases}$ and $q(t+3) = q(t)$

Since $p(t) = \cos 20\pi t$

$$= \cos 30\omega_0 t = \frac{1}{2} e^{j30\omega_0 t} + \frac{1}{2} e^{-j30\omega_0 t}$$

$\therefore\ a_{30} = \frac{1}{2}$, $a_{-30} = -\frac{1}{2}$.

ECE 301 Signals and Systems
Homework # 5 Solution

From Table 4.2

$$\boxed{x(t)=\begin{cases}1, & |t|<T_1\\ 0, & T_1<|t|\le\frac{T}{2}\end{cases} \text{ and } x(t+T)=x(t) \overset{F.S.}{\Longleftrightarrow} \frac{\sin k\omega_0 T_1}{k\pi}}$$

The Fourier Series coefficients of $q_1(t)$ are.

$$b_k = \frac{\sin k\frac{2\pi}{3}}{k\pi}$$

$$\therefore\ C_k = a_{30}b_{k-30} + a_{-30}b_{k+30}.$$

$$= \boxed{\frac{1}{2}\cdot\frac{\sin(k-30)\frac{2\pi}{3}}{k\pi} + \frac{1}{2}\frac{\sin(k+30)\frac{2\pi}{3}}{k\pi}}$$

ii) The period of the signal is 3. $\omega_0=\frac{2\pi}{T}=\frac{2\pi}{3}$.

Let $x_2(t)=p(t)q_2(t)$.

where $p(t)=\cos 20\pi t$, $q_2(t)=q_2'(t)+q_2''(t)$

where $q_2'(t)=\begin{cases}1, & 0<t<2\\ 0, & 2<t<3\end{cases}$ and $q_2'(t+3)=q_2'(t)$

$q_2''(t)=\begin{cases}1, & 0<t<1\\ 0, & 1<t<3\end{cases}$ and $q_2''(t+3)=q_2''(t)$.

From Table 4.2

$$\boxed{x(t)=\begin{cases}1, & |t|<T_1\\ 0, & T_1<|t|\le\frac{T}{2}\end{cases} \text{ and } x(t+T)=x(t) \overset{F.S.}{\Longleftrightarrow} \frac{\sin k\omega_0 T_1}{k\pi}}$$

Table 3.1

$$\boxed{x(t-t_0) \Longleftrightarrow a_k e^{-jk\omega_0 t_0} = a_k e^{-jk\frac{2\pi}{T}t_0}}$$

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$$b_k = b_k' + b_k''$$
$$= e^{-jk\frac{2\pi}{3}\cdot\frac{1}{2}} \cdot \frac{\sin k\frac{2\pi}{3}\cdot\frac{1}{2}}{k\pi} + e^{-j\frac{2\pi}{3}\cdot 1} \frac{\sin k\frac{2\pi}{3}\cdot 1}{k\pi}$$
$$= e^{-jk\frac{\pi}{3}} \cdot \frac{\sin k\frac{\pi}{3}}{k\pi} + e^{-jk\frac{2\pi}{3}} \cdot \frac{\sin k\frac{2\pi}{3}}{k\pi}.$$

$$C_k = a_{30} b_{k-30} + a_{-30} b_{k+30}$$
$$= e^{-j(k-30)\frac{\pi}{3}} \cdot \frac{\sin(k-30)\frac{\pi}{3}}{2k\pi} + e^{-j(k-30)\frac{2\pi}{3}} \cdot \frac{\sin(k-30)\frac{2\pi}{3}}{k\pi}$$
$$+ e^{-j(k+30)\frac{\pi}{3}} \frac{\sin(k+30)\frac{\pi}{3}}{k\pi} + e^{-j(k-30)\frac{2\pi}{3}} \cdot \frac{\sin(k+30)\frac{2\pi}{3}}{k\pi}$$

iii). The period of the signal is 4. $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$.

Let $x_3(t) = p(t) q_3(t)$

where $p(t) = \cos 20\pi t$, $q_3(t) = \begin{cases} e^{-|t|}, & |t|<1 \\ 0, & 1<|t|<2 \end{cases}$ and $q_3(t+4) = q(t)$.

The Fourier Series coefficients of $q_3(t)$ are

$$b_k = \frac{1}{4}\left[\int_{-1}^{0} e^t \cdot e^{-jk\omega_0 t} dt + \int_0^1 e^{-t} e^{-jk\omega_0 t} dt\right]$$
$$= \frac{1}{4}\left\{ \frac{1}{1-jk\omega_0} e^{(1-jk\omega_0)t}\Big|_{-1}^{0} - \frac{1}{1+jk\omega_0} e^{(-1-jk\omega_0)t}\Big|_0^1 \right\}$$
$$= \frac{1}{4}\left\{ \frac{1-e^{jk\omega_0 - 1}}{1-jk\omega_0} - \frac{e^{-jk\omega_0 - 1} - 1}{1+jk\omega_0} \right\}.$$

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$$= \frac{1}{4} \cdot \frac{(1-e^{jk\omega_0-1})(1+jk\omega_0) - (1-jk\omega_0)(e^{-1-jk\omega_0}-1)}{1+k^2\omega_0^2}$$

$$= \frac{e-\cos(k\omega_0)+k\omega_0\sin(k\omega_0)}{2e(1+k^2\omega_0^2)}$$

$$C_k = a_{30}b_{k-30} + a_{-30}b_{k+30}.$$

$$= \frac{e-\cos[(k-30)\frac{\pi}{2}]+(k-30)\frac{\pi}{2}\sin[(k-30)\frac{\pi}{2}]}{4e[1+(k-30)^2\cdot\frac{\pi^2}{4}]}$$

$$+ \frac{e-\cos[(k+30)\frac{\pi}{2}]+(k+30)\frac{\pi}{2}\sin[(k+30)\frac{\pi}{2}]}{4e[1+(k+30)^2\frac{\pi^2}{4}]}.$$

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Homework # 5 Solution

c) Let $z(t) = x(t) \cdot x^*(t)$

$$= |x(t)|^2 = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

From Table 3.1

$$\boxed{x^*(t) \overset{F.S.}{\Longleftrightarrow} b_k = a^*_{-k}}$$

$$\frac{1}{T_0}\int_0^{T_0} |x(t)|^2 dt = \frac{1}{T_0}\int_0^{T_0} z(t)\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0} \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t} dt$$

$$= \frac{1}{T_0}\left\{ C_0 T_0 + \left[\sum_{k\neq 0} C_k \cdot \frac{1}{jk\omega_0} \cdot e^{jk\omega_0 t}\right]\Bigg|_0^{T_0} \right\}$$

$$= C_0 .$$

$$C_0 = \sum_{n=-\infty}^{\infty} a_n b_{k-n}$$

$$= \sum_{n=-\infty}^{\infty} a_n a^*_{-k}$$

$$= \sum_{n=-\infty}^{\infty} |a_n|^2$$

$$\therefore \boxed{\frac{1}{T_0}\int_0^{T_0} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |a_n|^2}$$ Parseval's relation

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Homework # 5 Solution

4.21

a) $[e^{-at}\cos\omega_0 t]u(t)$, $a>0$.

$$X(j\omega) = \int_{-\infty}^{\infty} [e^{-at}\cos\omega_0 t]u(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^{\infty} [e^{-at} \cdot \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \cdot e^{-j\omega t}\, dt.$$

$$= \frac{1}{2}\int_0^{\infty} e^{[-a-j(\omega-\omega_0)]t}\, dt + \frac{1}{2}\int_0^{\infty} e^{[-a-j(\omega+\omega_0)]t}\, dt$$

$$= \frac{1}{2}\left.\frac{e^{[-a-j(\omega-\omega_0)]t}}{a+j(\omega-\omega_0)}\right|_0^{\infty} + \frac{1}{2}\cdot\left.\frac{e^{[-a-j(\omega+\omega_0)]t}}{a+j(\omega+\omega_0)}\right|_0^{\infty}$$

$$= \frac{1}{2}\cdot\frac{1}{a+j(\omega-\omega_0)} + \frac{1}{2}\cdot\frac{1}{a+j(\omega+\omega_0)}$$

Alternatively, you can use Table 4.2

$$\boxed{e^{-\alpha t}u(t) \overset{F.T.}{\Longleftrightarrow} \frac{1}{\alpha+j\omega}, \quad \alpha>0.}$$

$$\boxed{\cos\omega_0 t \overset{F.T.}{\Longleftrightarrow} \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]}$$

Thus, $[e^{-at}\cos\omega_0 t]u(t)$

$$= e^{-at}u(t)\cdot\cos\omega_0 t$$

$$\Updownarrow \text{ F.T.}$$

$$\frac{1}{2\pi}\cdot\frac{1}{a+j\omega} * \{\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]\}$$

$$= \frac{1}{2}\cdot\frac{1}{a+j(\omega-\omega_0)} + \frac{1}{2}\cdot\frac{1}{a+j(\omega+\omega_0)}$$

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Homework # 5 Solution

d) $\sum_{k=0}^{\infty} \alpha^k \delta(t-kT)$, $|\alpha|<1$.

From Table 4.1 $\boxed{\delta(t-t_0) \Longleftrightarrow e^{-j\omega t_0}}$

$$\sum_{k=0}^{\infty} \alpha^k \delta(t-kT) \stackrel{F.T.}{\Longleftrightarrow} \sum_{k=0}^{\infty} \alpha^k e^{-j\omega kT} = \sum_{k=0}^{\infty} (\alpha e^{-j\omega T})^k$$

$$= \frac{1}{1-\alpha e^{-j\omega T}}$$

f) From Table 4.2.

$$\boxed{\frac{\sin \pi t}{\pi t} \stackrel{F.T.}{\Longleftrightarrow} X(j\omega) = \begin{cases} 1, & |\omega|<\pi \\ 0, & |\omega|>\pi \end{cases}}$$

By the shifting property.

$$\frac{\sin 2\pi(t-1)}{\pi(t-1)} \stackrel{F.T.}{\Longleftrightarrow} X'(j\omega) = \begin{cases} e^{-j\omega} & , |\omega|<2\pi \\ 0 & , |\omega|>2\pi. \end{cases}$$

$$\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi t}\right] \stackrel{F.T}{\Longleftrightarrow} \frac{1}{2\pi} \cdot X(j\omega) * X'(j\omega) = Y(j\omega)$$

i) $\omega+\pi<-2\pi$, $\omega<-3\pi$

$Y(j\omega)=0$

ii) $-2\pi<\omega+\pi<0$, $-3\pi<\omega<-\pi$

$$Y(j\omega) = \frac{1}{2\pi}\int_{-2\pi}^{\omega+\pi} e^{-j\tau}\, d\omega.$$

$$= -\frac{j}{2\pi}(1+e^{-j\omega})$$

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iii) $0 < \omega + \pi < 2\pi$, $-\pi < \omega < \pi$.

$$Y(j\omega) = \frac{1}{2\pi} \int_{\omega-\pi}^{\omega+\pi} e^{-j\tau} \, d\tau$$

$$= \frac{1}{2\pi} \cdot (-2\sin\omega)$$

$$= -\frac{\sin\omega}{\pi}.$$

iv) $2\pi < \omega + \pi < 4\pi$, $\pi < \omega < 3\pi$.

$$Y(j\omega) = \frac{1}{2\pi} \cdot \int_{\omega-\pi}^{2\pi} e^{-j\tau} d\tau$$

$$= \frac{j}{2\pi} (1 + e^{-j\omega}).$$

v) $\omega - \pi > 2\pi$, $\omega > 3\pi$.

$$Y(j\omega) = 0.$$

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Homework # 5 Solution

i) $x(t) = \begin{cases} 1-t^2, & 0<t<1 \\ 0, & \text{otherwise.} \end{cases}$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

$$= \int_{0}^{1} (1-t^2)\, e^{-j\omega t}\, dt. \quad \circledast$$

$$= \boxed{\frac{1}{j\omega} - \frac{2e^{-j\omega}}{\omega^2} - \frac{2(e^{-j\omega}-1)}{j\omega^3}}$$

$\circledast$ You will need to do integration by parts twice.

4.22

b) $X(j\omega) = \cos(4\omega + \frac{\pi}{3})$

$$= \frac{e^{j(4\omega+\frac{\pi}{3})} + e^{-j(4\omega+\frac{\pi}{3})}}{2}$$

$$= \frac{e^{j\frac{\pi}{3}}}{2} \cdot e^{j4\omega} + \frac{e^{-j\frac{\pi}{3}}}{2} \cdot e^{-j4\omega}$$

From Table 4.2.

$$\boxed{\delta(t-t_0) \overset{F.T}{\Longleftrightarrow} e^{-j\omega t_0}}$$

$$x(t) = \frac{e^{j\frac{\pi}{3}}}{2}\, \delta(t+4) + \frac{e^{-j\frac{\pi}{3}}}{2}\, \delta(t-4).$$

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c) $X(j\omega) = \begin{cases} |\omega| e^{-3j\omega}, & |\omega| < 1 \\ 0, & |\omega| > 1 \end{cases}$

$$x(t) = \frac{1}{2\pi} \cdot \int_{-1}^{0} (-\omega) e^{-3j\omega} e^{j\omega t} d\omega + \frac{1}{2\pi} \int_{0}^{1} \omega e^{-3j\omega} e^{j\omega t} d\omega.$$

$$= \frac{1}{\pi} \left[\frac{\sin(t-3)}{t-3} + \frac{\cos(t-3) - 1}{(t-3)^2} \right].$$

d) $X(j\omega) = 2[\delta(\omega-1) - \delta(\omega+1)] + 3[\delta(\omega-2\pi) + \delta(\omega+2\pi)]$.

From Table 4.2.

$$\boxed{\cos\omega_0 t \overset{F.T}{\Longleftrightarrow} \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]}$$

$$\boxed{\sin\omega_0 t \overset{F.T}{\Longleftrightarrow} \frac{\pi}{j}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]}$$

we obtain.

$$\boxed{x(t) = \frac{2j}{\pi} \sin t + \frac{3}{\pi} \cos(2\pi t)}$$

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$\boxed{4.23}$

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & o. \end{cases}$$

$$X_0(j\omega) = \int_{-\infty}^{\infty} x_0(t) e^{-j\omega t} dt$$

$$= \int_0^1 e^{-t} e^{-j\omega t} dt$$

$$= \frac{1}{-1-j\omega} e^{(-1-j\omega)t} \Big|_0^1$$

$$= \frac{1-e^{-1-j\omega}}{1+j\omega}.$$

b) $x_2(t) = x_0(t) - x_0(-t)$

From Table 4.1

$$\boxed{x(-t) \Longleftrightarrow X(-j\omega)}$$

we obtain

$$\boxed{X_2(j\omega) = \frac{1-e^{-1-j\omega}}{1+j\omega} - \frac{1-e^{-1+j\omega}}{1-j\omega}}$$

c) $x_3(t) = x_0(t+1) + x_0(t)$.

From Table 4.1 $\boxed{x(t-t_0) \Longleftrightarrow e^{-j\omega t_0} X(j\omega)}$

$$\boxed{X_3(j\omega) = \frac{1-e^{-1-j\omega}}{1+j\omega} + e^{j\omega} \cdot \frac{1-e^{-1-j\omega}}{1+j\omega}.}$$

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Homework # 5 Solution

4.24

a) (1) $Re\{X(j\omega)\}=0$

$\Rightarrow X(j\omega) = -X^*(j\omega)$,

$\Rightarrow x(t) = -x^*(-t)$. ——— ①

For a real signal, ① becomes $x(t) = -x(-t)$, which implies that $x(t)$ is odd.

Obviously (a) and (d) satisfies the condition.

(2) $Im\{X(j\omega)\}=0$

$\Rightarrow X(j\omega) = X^*(j\omega)$

$\Rightarrow x(t) = x^*(-t)$ ——— ②

For a real signal, ② becomes $x(t) = x(-t)$, which implies that $x(t)$ is even.

Obviously, (e) and (f) satisfies the condition.

(3) From Table 4.1 we know that $e^{j\alpha\omega}X(j\omega)$ is the Fourier Transform of $x(t+\alpha)$.

If $e^{j\alpha\omega}X(j\omega)$ is real, from (2) we know that $x(t+\alpha)$ must be even. So the problem effectively asks whether there exists some real α that makes $x(t+\alpha)$ an even signal.

Obviously, (a) and (b) satisfy the condition.

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(4). $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw) e^{jwt} dw.$ (a)(d)(f) satisfy.

Let $t=0$. $x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)\, dw.$

$$\therefore \boxed{\int_{-\infty}^{\infty} X(jw) dw = 0 \Longleftrightarrow x(0)=0.}$$

(5) From Table 4.1 $\boxed{\frac{d}{dx}x(t) \Longleftrightarrow jw X(jw)}$

$$\therefore \frac{d}{dx}x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} jw X(jw) e^{jwt} dw.$$

(b)(c)(e)(f) satisfy.

Let $t=0$,

$$\frac{d}{dt}x(0) = \frac{j}{2\pi}\int_{-\infty}^{\infty} w X(jw)\, dw.$$

$$\therefore \boxed{\int_{-\infty}^{\infty} wX(jw) dw = 0 \Longleftrightarrow \frac{d}{dt}x(0) = 0}$$

(6) If $X(jw)$ is periodic, then it can be written in a Fourier Series representation,

$$X(jw) = \sum_{k=-\infty}^{\infty} A_k e^{jkt_0 w}, \quad t_0 = \frac{2\pi}{W}.\ W \text{ is the period of } X(jw)$$

From Table 4.2. $\boxed{\delta(t-t_0) \overset{F.T}{\Longleftrightarrow} e^{jwt_0}}$

$$X(jw) = \sum_{k=-\infty}^{\infty} A_k e^{jkt_0 w} \overset{F.T}{\Longleftrightarrow} x(t) = \sum_{k=-\infty}^{\infty} A_k \delta(t-kt_0)$$

It is seen that $x(t)$ is an impulse train with a period of $\frac{2\pi}{W}$.

~~Obviously none of the signals satisfies this.~~ **Signal (b) is of the required form.**

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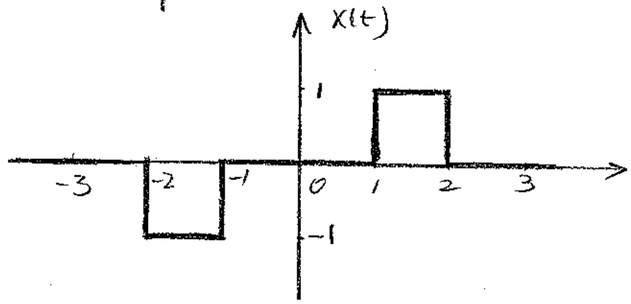
b) A real signal $x(t)$ that satisfies (1) (4) and (5) is such that

$$\begin{cases} x(t) \text{ is odd.} \\ x(0) = 0 \\ \frac{d}{dt}x(t)\Big|_{t=0} = 0 \end{cases}$$

One example could be.



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4.26 a)

$$x(t) = te^{-2t}u(t), \quad \overset{F.T}{\Longleftrightarrow} \quad X(j\omega) = \frac{1}{(2+j\omega)^2}$$

$$h(t) = te^{-4t}u(t) \quad \overset{F.T.}{\Longleftrightarrow} \quad H(j\omega) = \frac{1}{(4+j\omega)^2}$$

$$Y(j\omega) = \frac{1}{(2+j\omega)^2} \cdot \frac{1}{(4+j\omega)^2}$$

$$= \frac{1}{4(2+j\omega)} + \frac{1}{4(2+j\omega)^2} - \frac{1}{4(4+j\omega)} + \frac{1}{4(4+j\omega)^2}$$

$$\boxed{y(t) = \frac{1}{4}e^{-2t}u(t) + \frac{1}{4}te^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t) + \frac{1}{4}te^{-4t}u(t)}$$

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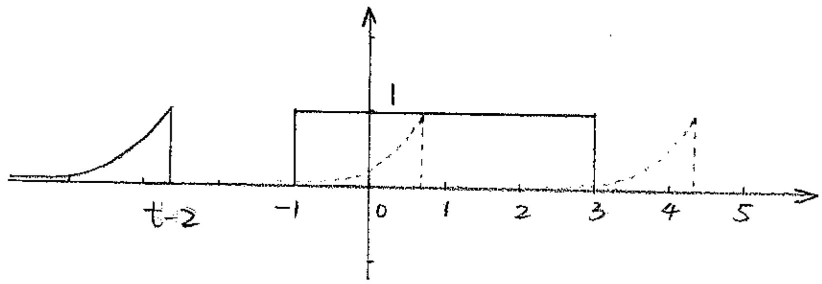
b) Time Domain Convolution



i) $t-2<-1$, $t<1$.

$$y(t)=0.$$

ii) $-1<t-2<3$, $1<t<5$

$$y(t)=\int_{-1}^{t-2} e^{(\tau-t+2)}\,d\tau$$

$$= 1-e^{1-t}$$

iii) $t-2>3$, $t>5$

$$y(t)=\int_{-1}^{3} e^{(\tau-t+2)}\,d\tau$$

$$= e^{(5-t)}-e^{(1-t)}$$

Frequency Domain Multiplication

$$x(t) \overset{F.T.}{\Longleftrightarrow} X(j\omega)=\frac{e^{-j2\omega}}{1+j\omega}.$$

$$h(t) \overset{F.T.}{\Longleftrightarrow} H(j\omega)=e^{-j\omega}\cdot\frac{2\sin 2\omega}{\omega}$$

$$Y(j\omega)=H(j\omega)X(j\omega)$$

$$=\frac{e^{j\omega}}{j\omega}-\frac{e^{-j5\omega}}{j\omega}-\frac{e^{-j\omega}}{1+j\omega}+\frac{e^{-j5\omega}}{1+j\omega}$$

ECE 301 Signals and Systems
Homework # 5 Solution

$$= (e^{-j\omega} - e^{-j5\omega})\left[\frac{1}{j\omega} + \pi\delta(\omega)\right] - \frac{1}{2}(e^{-j\omega} - e^{-j5\omega})2\pi\delta(\omega)$$

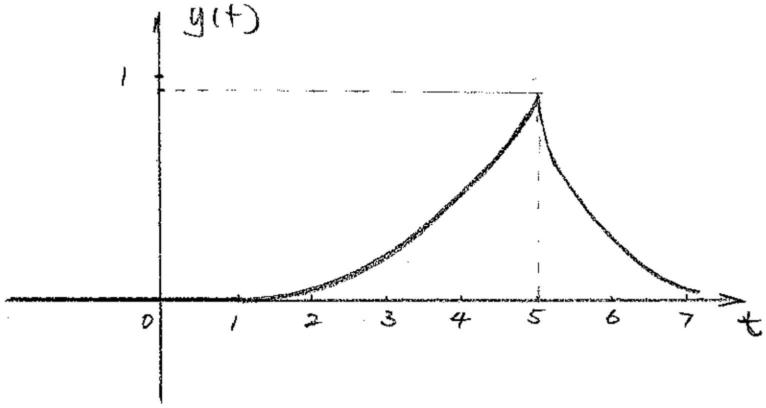
$$- (e^{-j\omega} - e^{-j5\omega}) \cdot \frac{1}{1+j\omega}.$$

The second term $\frac{1}{2}(e^{-j\omega} - e^{-j5\omega})2\pi\delta(\omega)$ is effectively 0.

$e^{-j\omega}$ and $e^{-j5\omega}$ each correspond to a time shift.

$$Y(j\omega) \overset{F.T.}{\Longleftrightarrow} y'(t) = u(t-1) - u(t-5) - \left[e^{-(t-1)}u(t-1) - e^{-(t-5)}u(t-5)\right]$$

It can be shown that $y'(t) = y(t)$.



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$\boxed{4.28}$ a) From Table 4.2 $\boxed{e^{j\omega_0 t} \overset{F.T.}{\Longleftrightarrow} 2\pi\delta(\omega-\omega_0)}$

$$P(j\omega) = \sum_{n=-\infty}^{\infty} a_n \cdot 2\pi\,\delta(\omega - n\omega_0)$$

$$\because\ y(t) = x(t)\cdot p(t) \overset{F.T.}{\Longleftrightarrow} \frac{1}{2\pi} X(j\omega)\cdot P(j\omega)$$

$$= \frac{1}{2\pi} X(j\omega)\cdot \sum_{n=-\infty}^{\infty} a_n \cdot 2\pi\,\delta(\omega - n\omega_0)$$

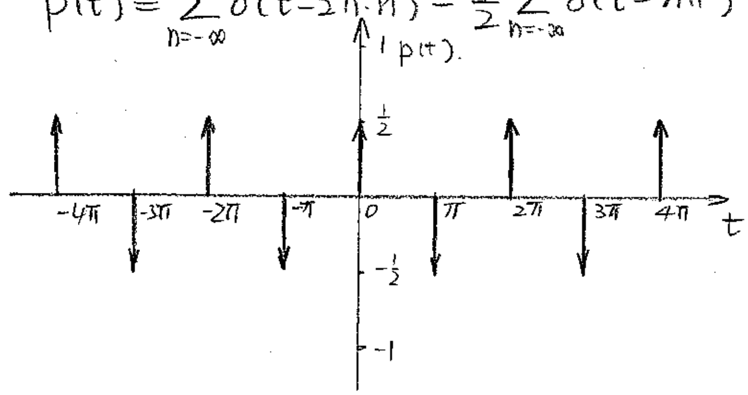
$$= \sum_{n=-\infty}^{\infty} a_n X[j(\omega - n\omega_0)].$$

b) iii) $p(t) = \cos(2t) = \dfrac{e^{j2t} + e^{-j2t}}{2}$

$\therefore\ a_1 = a_{-1} = \frac{1}{2}$, $\omega_0 = 2$.

$\therefore\ Y(j\omega) = \frac{1}{2} X[j(\omega-2)] + \frac{1}{2} X[j(\omega+2)]$.

ix) $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - 2\pi\cdot n) - \frac{1}{2}\sum_{n=-\infty}^{\infty} \delta(t - n\pi)$



$$a_k = \int_{-\varepsilon}^{2\pi-\varepsilon} \frac{1}{2}[\delta(t) - \delta(t-\pi)]\, e^{-jkt}\, dt.$$

$$= \frac{1}{2}(1 - e^{-jk\pi}) = \frac{1-(-1)^k}{2}$$

$$Y(j\omega) = \sum_{n=-\infty}^{\infty} a_n X[j(\omega - n)]. \quad = \frac{1}{2}\sum_{n=-\infty}^{\infty} X[j(\omega - 2n - 1)]$$

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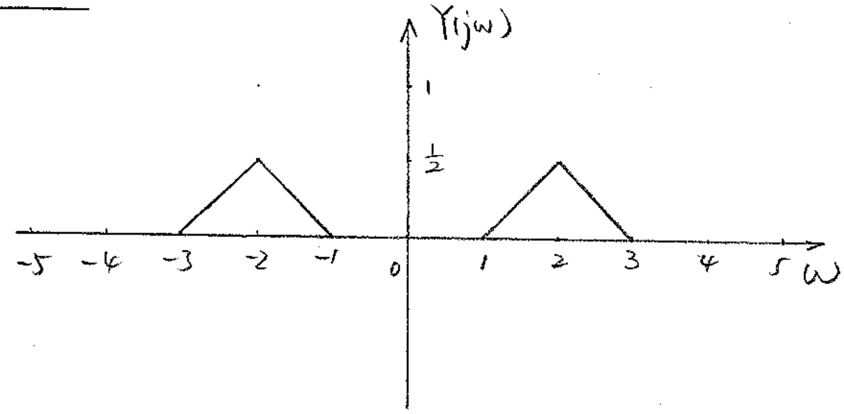
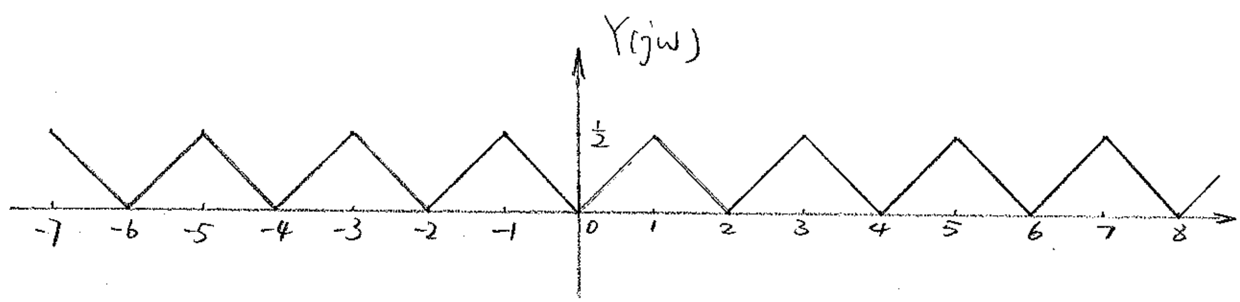
Spectrum

iii)



ix)



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b)

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

$$\Updownarrow \text{F.T.}$$

$$(j\omega)^2 Y(j\omega) + 6j\omega Y(j\omega) + 8Y(j\omega) = 2X(j\omega)$$

$$Y(j\omega) = \frac{2}{(j\omega)^2 + 6j\omega + 8} \cdot X(j\omega)$$

$$X(j\omega) = \frac{1}{(2+j\omega)^2}$$

$$\therefore\ Y(j\omega) = \frac{2}{(2+j\omega)(4+j\omega)} \cdot \frac{1}{(2+j\omega)^2}$$

$$= \frac{2}{(4+j\omega)(2+j\omega)^3}$$

$$= -\frac{1}{4(4+j\omega)} + \frac{1}{4(2+j\omega)} - \frac{1}{2(2+j\omega)^2} + \frac{1}{(2+j\omega)^3}$$

$$\Updownarrow \text{F.T.}$$

$$\boxed{y(t) = -\frac{1}{4}e^{-4t}u(t) + \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + \frac{t^2}{2}e^{-2t}u(t)}$$