1.6 Basic Systems Properties

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4:11 PM

memoryless / with memory

memoryless → if for any time t_0 the output at t_0 depends only on the input at t_0 $(x(t_0))$

ex. $y(t) = x(t) + x(t-1)$ is not memoryless

$y(t) = 2x(t)$ is memoryless

$y(t) = x(t+1)$ not

$y(t) = (t-1)\,x(t)$ is memoryless

$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$ is not

Invertable

$y(t) = 2x(t) + 3$ - invertable

$y(t) = x(t)^2$ - is not — you loose the sign info, not distinct outputs

To invert $y(t) = 2x(t)+3$

① isolate $x(t)$: $x(t) = \frac{y(t)-3}{2}$

② swap x & y ⇒ $y(t) = \frac{x(t)-3}{2}$

Def: An invertable system is a system such that distinct inputs yield distinct outputs

Ex. $y[n] = x[n+7]$ invertable? - yes

$y[n] = x[n-7]$

Causality

Def A system is causal if the output only depends on the input at the present time & past time. In other words the output at t_0 is dependent on $t = t_0$ or $t < t_0$ only.

$y(t) = x(t+1)$ - not causal

$y(t) = 2$ - causal

$y(t) = \int_t^{\infty} x(t)$

Stability

Def: A system is stable if bounded inputs yield bounded outputs, i.e. if $x(t)$ is bounded, then $y(t)$ is also bounded.

$\exists$ $\varepsilon > 0$

exist

Time Invariance

Def 1: A system is called time invariant (TI) if for any input signal $x(t)$ and for any time t_0, the output to the shifted input $x(t-t_0)$ is the shifted output $y(t-t_0)$.

Def 2: If $x(t) \to \boxed{sys} \to y(t)$

then

$x(t-t_0) \to \boxed{sys} \to y(t-t_0)$ for any $t_0 \in \mathbb{R}$

Def 3: If the cascade $x(t) \to \boxed{\text{time delay by } t_0} \to \boxed{\text{system}} \to y(t)$

yields the same output of $x(t) \to \boxed{sys} \to \boxed{delay} \to y(t)$

Def 4: If the system commute with a time delay.

Ex 1. $y(t) = 10\,x(t)$ TI

$y(t) = (x(t))^2$ $\rightarrow$ $x(t) \rightarrow \boxed{TD} \rightarrow y(t) = x(t-t_0)$

$\rightarrow \boxed{sys} \rightarrow z(t) = y(t)^2$

$= x(t-t_0)^2$

$x(t) \rightarrow \boxed{sys} \rightarrow y(t) = (x(t))^2 \rightarrow \boxed{TD}$ $z(t) = y(t-t_0)$

$\|$

$y(\cancel{t}) = (x(\cancel{t}))^2$ so $(x(t-t_0))^2$

<u>yes</u> is TI

$y(t) = t\,x(t)$

$x(t) \rightarrow \boxed{TD} = y(t) = x(t-t_0) \rightarrow \boxed{sys} \rightarrow z(t) = t\,y(t)$

$= t\,x(t-t_0)$

$x(t) \rightarrow \boxed{sys} = y(t) = t\,x(t) \rightarrow \boxed{TD} \rightarrow z(t) = y(t-t_0)$

$= (t-t_0)\,x(t-t_0)$

$\neq$ Not TI

Linearity

Def 1: A system is "linear" if for any combination $a, b, \in \mathbb{C}$ and for any inputs $x_1(t)$, $x_2(t)$ yielding $y_1(t)$ and $y_2(t)$ the system's response to $a x_1(t) + b x_2(t)$ is $a y_1(t) + b x_2(t)$

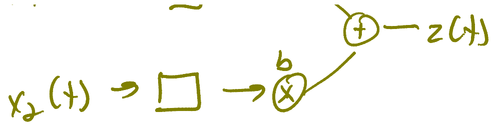
Def 2: If $x_1(t) \rightarrow \boxed{} \rightarrow y_1(t)$

$x_2(t) \rightarrow \boxed{} \rightarrow y_2(t)$

$\Rightarrow a\, x_1(t) + b\, x_2(t) \rightarrow \boxed{} \rightarrow a y_1(t) + b y_2(t)$

$\forall a, b \in \mathbb{C}$

Def 3: $x_1(t) \rightarrow \boxed{} \rightarrow \otimes$ a



yields same
output as