

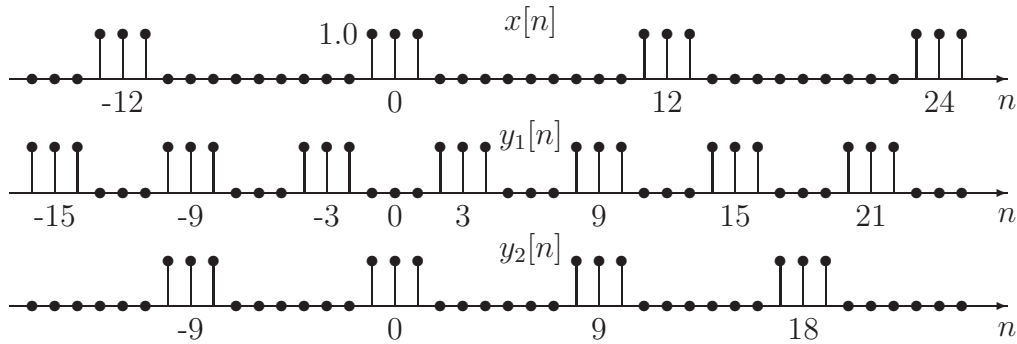
Name: \_\_\_\_\_

**General Instructions:**

- The exam is closed book and closed notes. Calculators are **not** allowed or needed.
- A formula sheet will be handed out.
- Your work must be explained to receive full credit.
- Point values for each problem are as indicated. The exam totals 100 points.
- Do all work in the blue books provided. Put your name and student identification number on the blue book. This exam problem sheet **must be handed in** with your blue book. You may keep the formula sheet.
- All plots must be carefully drawn with axes labeled.
- If you finish the exam during the first 50 minutes, you may turn it in and leave. During the last 10 minutes you must remain seated until we pick up exams and blue books from everyone.
- This exam is for Krogmeier's section only.

**Do not open the exam until you are told to begin.**

1. (25 pts.) Consider the periodic discrete-time signals  $x[n]$ ,  $y_1[n]$ , and  $y_2[n]$  shown below.
- Find the discrete-time Fourier series for  $x[n]$  and simplify. Note that you do not need the answer to this part for the rest of the problem.
  - For  $x[n]$  and  $y_1[n]$ :
    - Does there exist a LTI system with input  $x[n]$  and output  $y_1[n]$ ?
    - If the answer to the previous question is “yes,” determine if there is more than one such LTI system. Also, what is the relationship between the frequency response  $H(e^{j\omega})$  of such an LTI system and the discrete-time Fourier series coefficients of  $x[n]$  and  $y_1[n]$ ?
  - For  $x[n]$  and  $y_2[n]$ :
    - Does there exist a LTI system with input  $x[n]$  and output  $y_2[n]$ ?
    - If the answer to the previous question is “yes,” determine if there is more than one such LTI system. Also, what is the relationship between the frequency response  $H(e^{j\omega})$  of such an LTI system and the discrete-time Fourier series coefficients of  $x[n]$  and  $y_2[n]$ ?



2. (25 pts.) The following questions concern the discrete-time Fourier transform (DTFT).

(a) Find the DTFT  $F(e^{j\omega})$  of the following sequence and simplify

$$f[n] = e^{-3n} \cos(3n)u[n].$$

(b) Find the inverse DTFT  $x[n]$  of the following

$$X(e^{j\omega}) = \frac{e^{-j2\omega} - 1}{e^{-j2\omega} - 4}.$$

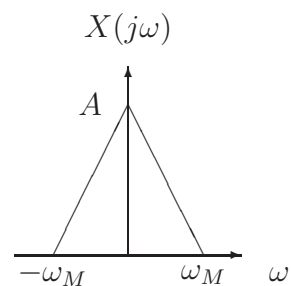
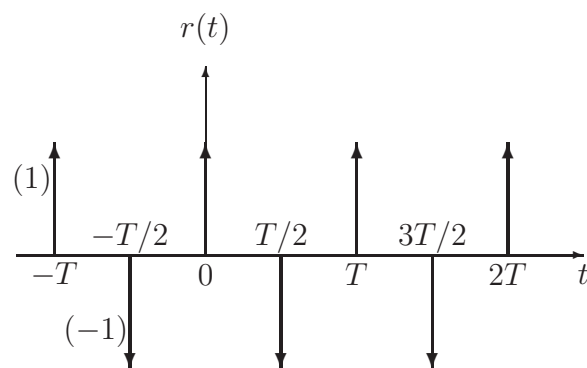
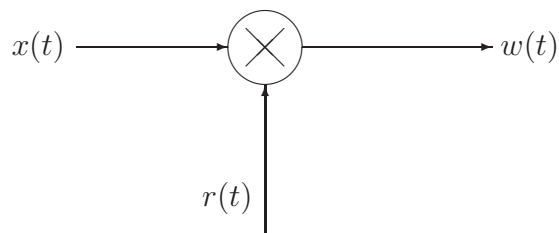
(c) Find the energy in the signal having DTFT

$$Y(e^{j\omega}) = \frac{1 - ae^{-j\omega}}{a - e^{-j\omega}}$$

where  $-1 < a < 1$ . *Hint:* The magnitude of the DTFT has a particularly simple form.

3. (25 pts.) The following questions refer to the impulse sampling scheme shown below.

- Find and sketch the Fourier transform  $R(j\omega)$  of the periodic pulse train  $r(t)$ .
- Find and sketch the Fourier transform  $W(j\omega)$  of the weighted impulse train  $w(t)$ . Draw your sketch for the case where there is no aliasing.
- What is the condition relating  $T$  and  $\omega_M$ , which must be satisfied so that there is no aliasing?



4. (25 pts.) Let  $x(t)$  be a real-valued signal for which  $X(j\omega) = 0$  when  $|\omega| > 1000\pi$ . Amplitude modulation is performed to produce the signal

$$g(t) = x(t) \cos(1000\pi t).$$

Assume the spectrum of  $x(t)$  is real-valued and as given in the figure.

- Find and plot the real and imaginary parts of  $G(j\omega)$ , the spectrum of  $g(t)$ .
- Find and plot the real and imaginary parts of  $W(j\omega)$ , the spectrum of  $w(t)$ .
- Find  $y(t)$ , the output of the demodulator in the figure assuming that the low pass filter (LPF) is ideal with a cutoff frequency of  $1000\pi$  and a passband gain of 2.

