

1. Let  $a_n = \frac{-1}{n}$ , for  $n \in \mathbb{N}$ .
  - (a) Prove  $\sum_1^\infty a_n$  converges, but not absolutely (can you do this without the " $p$ -test"?)
  - (b) Prove there is a rearrangement,  $\{a_{n_k}\}$  such that  $\sum_1^\infty a_{n_k}$  converges to .24569004 (without using the theorem of Riemann in the text) and explicitly give  $a_{n_1}, a_{n_2}, \dots, a_{n_{10}}$ .
2. The following gives a way to use basic calculus to prove the " $p$  - test", namely that  $\sum \frac{1}{n^p}$  converges exactly when  $p > 1$ . Draw rectangles,  $R_n$  in the plane with base  $[n, n + 1]$  (on the  $X$ - axis) and height  $\frac{1}{n^p}$ . Dominate them with a function, and integrate the function from say  $[2, \infty)$ . Fill in the details of this proof.
3.
  - (a) Let  $C$  denote the Cantor set, and notice  $C^c$  is a countable union of disjoint intervals, so it's length is defined. Find the length of  $C^c$ .
  - (b) Now repeat the construction of the Cantor set, but cut out the middle fourths. What is the length of its complement?
  - (c) Given any  $r \in [0, 1)$  does there exist a Cantor set whose complement has length  $r$ ?
4. Prove that the Cantor set is  $\{r \in [0, 1]; r\text{'s base 3 decimal contains no } 1\}$ . Warning:  $\frac{1}{3} = .1_{\text{base } 3} = .0\overline{22}_{\text{base } 3}$ . You need to be explicit about the decimal representation you choose in the ambiguous cases for the statement above to make sense.
5. Repeat the construction of the Cantor set, but now cut out the middle third *closed* interval. Call this new set  $\mathcal{C}$ .
  - (a) Is  $\mathcal{C}$  empty?
  - (b) Is  $\mathcal{C}$  open? closed?
  - (c) What is the cardinality of  $\mathcal{C}$ ?
6. Let  $X$  be a normed linear space (i.e. a normed vector space over, say  $\mathbb{R}$ ). Show  $X$  is complete  $\Leftrightarrow$  every absolutely convergent series of  $X$  converges in  $X$ .