(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No	
If $y(t) = x(2t)$, is the system causal?	☐	☒	ca
If $y(t) = (t+2)x(t)$, is the system causal?	☒	☐	
If $y(t) = x(-t^2)$, is the system causal?	☒	☐	X
If $y(t) = x(t) + t - 1$, is the system memoryless?	☐	☒	X
If $y(t) = x(t^2)$, is the system memoryless?	☐	☒	
If $y(t) = x(t/3)$, is the system stable?	☒	☐	
If $y(t) = tx(t/3)$, is the system stable?	☒	☐	X
If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable?	☐	☒	
If $y(t) = \sin(x(t))$, is the system time invariant?	☒	☐	
If $y(t) = u(t) * x(t)$, is the system LTI?	☒	☐	
If $y(t) = (tu(t)) * x(t)$, is the system linear?	☐	☒	X

$x(t) \to \boxed{ID} \to \boxed{sys} \to z(t) = \sin(y(t))$

$y(t) = x(t-t_0) \qquad = \sin(x(t-t_0))$

$x(t) \to \boxed{sys} \to \boxed{ID} \to z(t) = y(t-t_0)$

$y(t) = \sin(x(t)) \qquad \sin(x(t-t_0))$

12

(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\,u(t+2-\tau)\,d\tau$$

$$= \int_{0}^{\infty} e^{-\tau}u(t+2-\tau)\,d\tau$$

$u(t+2-\tau) = 1$ when $t+2-\tau \geq 0$, $\tau \leq t+2$; else $u(t+2-\tau) = 0$

$$= \begin{cases} \int_{0}^{t+2} e^{-\tau}\,d\tau, & \cancel{t>0} \\ 0, & \text{else} \end{cases}$$

$$= \begin{cases} -1\left(e^{-\tau}\Big|_{0}^{t+2}\right) & t>0 \\ 0, & \text{else} \end{cases} = \begin{cases} -\left(e^{-(t+2)} - e^{0}\right), & t>0 \\ 0, & \text{else} \end{cases}$$

$$= \begin{cases} 1 - e^{-t-2} & t>0 \\ 0, & \text{else} \end{cases}$$

$|1+j| = \sqrt{(1+j)(1-j)} = \sqrt{1-j^2}$

$= \sqrt{1-(\sqrt{-1})^2} = \sqrt{1-(-1)} = \sqrt{2}$

15

$|e^{jt}| = 1$

$|1+j| = \sqrt{2}$

(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left|\frac{3e^{jt}}{1+j}\right|^2 dt = \int_{-\infty}^{\infty} \left(\frac{3(1)}{\sqrt{2}}\right)^2 dt$$

$$= \frac{9}{2} \int_{-\infty}^{\infty} dt = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left(\frac{3}{\sqrt{2}}\right)^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(\frac{9}{2}\right)(T-(-T)) = \lim_{T\to\infty} \left(\frac{1}{\cancel{2T}}\right)\left(\frac{9}{2}\right)(\cancel{2T})$$

$$= \frac{9}{2}$$

3

(15 pts) **4.** Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$

(Simplify your answer as much as possible.)

$$a_k = \frac{1}{T} \int^{T} x(t) e^{-jk(\frac{2\pi}{T})t} dt$$

$$= \frac{1}{4} \left[\int_0^2 \sin(t\pi) e^{-jk(\pi/2)t} dt + \int_2^4 0 \, dt \right]$$

$$= \frac{1}{4} \int_0^2 \sin(\pi t) e^{-jk(\pi/2)t} dt$$

5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

$$\begin{array}{rcl} \text{input} & & \text{output} \\ x_0[n] = \delta[n] & \rightarrow & y_0[n] = \delta[n-1], \\ x_1[n] = \delta[n-1] & \rightarrow & y_1[n] = 4\delta[n-2], \\ x_2[n] = \delta[n-2] & \rightarrow & y_2[n] = 9\delta[n-3], \\ x_3[n] = \delta[n-3] & \rightarrow & y_3[n] = 16\delta[n-4], \\ & \vdots & \\ x_k[n] = \delta[n-k] & \rightarrow & y_k[n] = (k+1)^2\delta[n-(k+1)] \text{ for any integer k.} \end{array}$$

(10 pts) **a)** Can this system be time-invariant? Explain.

Yes, this system can be TI because the $x[n] \rightarrow \boxed{TD} \rightarrow \boxed{X_n} \rightarrow z[n]$ produces the same result as $x[n] \rightarrow \boxed{X_n} \rightarrow \boxed{TD} \rightarrow z[n]$

0

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?

$u[n] = \sum_{k=0}^{N} \delta[n-k]$

$y[n] = u[n-1]$
$= \delta[n-k-1]$

5