Homework-1, ECE302

Solutions

(1) Given $S = \{1, 3, 5, 7, 9, 11\}$

$A = \{1, 3, 5\}$

$B = \{7, 9, 11\}$

$C = \{3, 5, 9, 11\}$

(a) $A \cup B = \{1, 3, 5, 7, 9, 11\}$

(b) $\overline{A \cap C} = \{1, 7, 9, 11\}$

(c) $\overline{A} \cup \overline{C} = \overline{A \cap C} = \{1, 7, 9, 11\}$

(d) $(A \cap C) \cap B = \phi$ since $A \cap B = \phi$

(e) $D = \{1\} = (\overline{A \cap C}) - B$

(2) (a) $S = \{HHH, HTT, HTH, HHT, THH, THT, TTH, TTT\}$

$A = \{HHH, HTT, THT, TTH\}$

~~(b) Let (a,b) denote the ordered pair of roll on the pair ~~number of heads and tails~~~~

~~$S = \{(a,b) \mid 1 \leq a,b \leq 6 \text{ and } a,b \in \mathbb{N}\}$~~

~~$A = \{(a,b) \mid 1 \leq a,b \leq 6 \text{ and } a \in \{2,4,6\} \text{ and } a,b \in \mathbb{N}\}$~~

(c) $S = \{0, 1, 2, 3, 4, \ldots, \infty\}$

$A = \{0, 1, 2, 3, 4, 5, 11, 12, 13, \ldots, \infty\}$

(d) Let (a,b) denote the ordered pair of the roll on the pair of dice

$S = \{(a,b) \mid 1 \leq a,b \leq 6 \text{ and } a,b \in \mathbb{N}\}$

$A = \{(a,b) \mid 1 \leq a,b \leq 6 \text{ and } a+b \text{ is even and } a,b \in \mathbb{N}\}$

(b) Let (a,b) denote the ordered pair of number of heads and tails

$S = \{(a,b) \mid 0 \leq a,b \leq 3 \text{ and } a+b=3 \text{ and } a,b \in \mathbb{W}\}$

$A = \{(a,b) \mid 0 \leq a,b \leq 3 \text{ and } a+b=3 \text{ and } a \text{ is odd and } a,b \in \mathbb{W}\}$

(3) (a) Total outcomes in sample space = 36

odds in favor = 18

$$\text{Probability} = \frac{18}{36} = 0.5$$

(b) Same as above.

(c) $\text{Probability} = \frac{27}{36} = 0.75$

(4) (a) P(transistor is defective) = $P(A)\,P(def/A) + P(B)\,P(def/B) + P(C)\,P(def/C)$

where $P(A)$ = Prob. of choosing a transistor from A

$P(def/A)$ = Prob. of the transistor being defective given its from A

$$P(\text{defective}) = \frac{1}{3}(0.05) + \frac{1}{3}(0.1) + \frac{1}{3}(0.25) = \frac{2}{15}$$

(b) From above; $P(\text{not defective}) = 1 - \frac{2}{15} = \frac{13}{15}$

$$P(C/\text{not def}) = \frac{P(\text{not def}|C)\cdot P(C)}{P(\text{not def})}$$

$$= \frac{0.75 \cdot \frac{1}{3}}{\frac{13}{15}} = 0.288$$

(5) (a) $P(A \cup B) = P(A^c \cap B) + P(A \cap B) + P(A \cap B^c) = P(A^c \cap B) + P(A \cap B) + P(A \cap B^c) + P(A \cap B) - P(A \cap B)$

$= P(A) + P(B) - P(A \cap B)$

(b) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$ as $A \subset B$, $\frac{P(A)}{P(B)} \geq P(A)$ as $P(B) > 0$ and ≤ 1

(c) $P(A/B\cap C)\cdot P(B|C)\cdot P(C) = \frac{P(A\cap B\cap C)}{\cancel{P(B\cap C)}} \cdot \frac{\cancel{P(B\cap C)}}{\cancel{P(C)}} \cdot \cancel{P(C)} = P(A\cap B\cap C)$