1. Prove or disprove: if f_n, g_n are uniformly convergent sequences on a set E then $f_n g_n$is also.

2. Let $C(K)\{f : K \to \mathbb{R} : f$ is continuous$\}$ and endow $C(K)$ with the sup norm, i.e.
$$\|f\| := \max_K |f|.$$

   (a) Show $\|\_\|$ is a norm, i.e. $d(f, g) := \|f - g\|$ is a metric, and given a constant, c, we have $\|cf\| = |c|\|f\|$.

   Definition: We say $f_n \to f$ in $C(K) \iff f_n, f \in C(K)$ and $\|fn - f\| \to 0$.

   (b) Show $C(K)$ is complete .

   (c) Set $K = [0, 1]$ and show closed and bounded is not equivalent to compact in $C[0, 1]$.

   (d) What other conditions do you need? An easy way to remember Arzela-Ascoli's theorem is that is gives sufficient conditions for compactness of subsets of $C[0, 1]$. Prove this; namely,

   Show that if $A \subset C[0, 1]$ is closed, bounded (both in the sup norm), and equicontinuous, A is compact.

   (e) (Another example). Let $A := \{f_n(x) = \dfrac{1}{nx + 1}\}$, and show it is closed (every point is isolated!) and bounded but not compact (hence not equicontinuous!)).

3. Let $f_n \to f$ uniformly, with allf_n and f differentiable on $[0, 1]$ (with the usual left and right hand limits for derivatives at 0 and 1 respectively). True or false:

   (a) $\int f_n \to \int f$.

   (b) $f_n' \to f'$

   (c) Show (b) holds if we also assume f_n' converges uniformly to a continuous g.