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(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No	
If $y(t) = x(2t)$, is the system causal? eg $t=1$ $y(1)=x(2)$ (future)	✓	✓	X
If $y(t) = (t+2)x(t)$, is the system causal?	✓		
If $y(t) = x(-t^2)$, is the system causal?		✓	
If $y(t) = x(t) + t - 1$, is the system memoryless?	✓		
If $y(t) = x(t^2)$, is the system memoryless?		✓	
If $y(t) = x(t/3)$, is the system stable?	✓		
If $y(t) = tx(t/3)$, is the system stable?	✓	✓	X
If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable?		✓	
If $y(t) = \sin(x(t))$, is the system time invariant?	✓	✓	X
If $y(t) = u(t) * x(t)$, is the system LTI?	✓	✓	X
If $y(t) = (tu(t)) * x(t)$, is the system linear?	✓		

$x(t) \to \boxed{sys} \to \sin(\square) \to \boxed{TD} \to (t-t_0) \to \sin(t-t_0)$

$x(t) \to \boxed{TD} \to (\square - t_0) \to \boxed{sys} \to \sin(t) \to \sin(t) - t_0$

$x(t) \to \boxed{sys} \to u(\square) * x(\square) \to \boxed{TD} \to (t-t_0) \to u(t-t_0) * x(t-t_0)$

$x(t) \to \boxed{TD} \to (\square - t_0) \to \boxed{sys} \to u(t) * x(t) \to (u(t) * x(t)) - t_0$

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$\star$ Question On Final $\star$

(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, u(t+2-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, u(t+2-\tau)\, d\tau$$

$u(\tau) = 0$
$\tau \leq 0$

$$\int_{0}^{\infty} e^{-\tau} u(t+2-\tau)\, d\tau$$

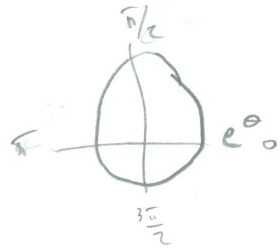
$u(t+2-\tau) = 0$
$t+2-\tau \leq 0$
$t+2 \leq \tau$

$$= \begin{cases} \int_0^{t+2} e^{-\tau}\, d\tau & t \geq -2 \\ 0 & \text{else} \end{cases}$$

$$= \left(\int_0^{t+2} e^{-\tau}\, d\tau\right) u(t+2)$$

$$= \left(-e^{-\tau}\Big|_0^{t+2}\right) u(t+2)$$

$$= \boxed{\left(-e^{-t-2} - 1\right) u(t+2)}$$



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(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$. $= \left(\frac{1}{1+j}\right)\left(3e^{jt}\right)$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} \left| \frac{1}{1+j}(3e^{jt}) \right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left| \frac{3}{1+j} \right|^2 dt$$

$$= \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| \frac{1}{1+j}(3e^{jt}) \right|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| \frac{3}{1+j} \right|^2 dt$$

$$= 3$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \frac{9}{2} \, dt$$

$$= \lim_{T\to\infty} \cancel{\frac{1}{2T}} \left(\frac{9}{2} \cancel{2T} \right)$$

$$\boxed{= \frac{9}{2}}$$

$$|x(t)| = \left| \frac{3e^{jt}}{1+j} \right|$$

$$= \frac{3|e^{jt}|}{|1+j|} = \frac{3(1)}{\sqrt{2}}$$

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3.22c Hw Problem

$\cos(\omega_0 t) = \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$

$\sin(\omega_0 t) = \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$

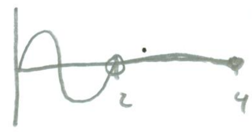
(15 pts) **4.** Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$

(Simplify your answer as much as possible.)

$T = 4 = k\frac{2\pi}{\omega_0}$

$\omega_0 = \frac{\pi}{2}$



$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\left(\frac{2\pi}{T}\right)t}\, dt \qquad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t}$$

$T = 4$

$$= \frac{1}{4}\int_0^4 x(t)\, e^{-jk\left(\frac{2\pi}{4}\right)t}\, dt$$

$$= \frac{1}{4}\int_0^2 \sin(\pi t)\, e^{-jk\left(\frac{\pi}{2}\right)t}\, dt + \cancel{\frac{1}{4}\int_2^4 0\, e^{-jk\left(\frac{\pi}{2}\right)t}\, dt}$$

$$= \frac{1}{4}\int_0^2 \sin(\pi t)\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{4}\int_0^2 \left(\frac{1}{2j}\left(e^{jk\frac{2\pi}{4}t} - e^{-jk\frac{2\pi}{4}t}\right)\right) e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{8j}\int_0^2 \left(e^{jk2t} - e^{-jk2t}\right) e^{-jk\frac{\pi}{2}t}$$

$$a_k = \frac{1}{4}\int_0^4 x(t)\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{2}\int_0^2 \sin \pi t\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{4}\int_0^2 \frac{e^{j\pi t} - e^{-j\pi t}}{2j}\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{8j}\int e^{j\pi\left(1-\frac{k}{2}\right)t} - e^{-j\pi\left(1+\frac{k}{2}\right)t}\, dt$$

if $k \neq 2, -2$

$$= \frac{1}{8j}\left[\frac{e^{j\pi\left(1-\frac{k}{2}\right)t}}{j\pi\left(1-\frac{k}{2}\right)} - \frac{e^{-j\pi\left(1+\frac{k}{2}\right)t}}{j\pi\left(1+\frac{k}{2}\right)}\right]_0^2$$

$$= \frac{-1}{8\pi}\left[\frac{e^{2j\pi\left(1-\frac{k}{2}\right)} - 1}{\left(1-\frac{k}{2}\right)} - \frac{e^{-2j\pi\left(1+\frac{k}{2}\right)} - 1}{\left(1+\frac{k}{2}\right)}\right]$$

$$= \frac{-1}{4\pi}\left[\frac{e^{2j\pi}\, e^{-jk\pi} - 1}{2-k} - \frac{e^{-2j\pi}\, e^{-jk\pi} - 1}{2+k}\right]$$

$$= \frac{-1}{4\pi}\left[\frac{(-1)^k - 1}{2-k} - \frac{(-1)^k - 1}{2+k}\right]$$

$$= \frac{1-(-1)^k}{4\pi}\left[\frac{1}{2-k} - \frac{1}{2+k}\right]$$

$$= \frac{1-(-1)^k}{4\pi}\left(\frac{4}{4-k^2}\right)$$

$$\boxed{a_k = \frac{1-(-1)^k}{\pi(4-k^2)}}$$

$$a_{-2} = a_2 = \frac{1}{8j}\left(\int_0^2 dt - \int_0^2 e^{-2j\pi t}\, dt\right)$$

$$= \frac{1}{8j}\left(2 - \frac{e^{-2j\pi t}}{-2j\pi}\Big|_0^2\right)$$

$a_0 = 0$

$$= \frac{1}{8j}\left(2 - \left(\frac{e^{-4j\pi} - 1}{-2j\pi}\right)\right)$$

$$\boxed{a_{-2} = a_2 = 1/4j}$$

5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

$$\begin{array}{rcl} \text{input} & & \text{output} \\ x_0[n] = \delta[n] & \rightarrow & y_0[n] = \delta[n-1], \\ x_1[n] = \delta[n-1] & \rightarrow & y_1[n] = 4\delta[n-2], \\ x_2[n] = \delta[n-2] & \rightarrow & y_2[n] = 9\delta[n-3], \\ x_3[n] = \delta[n-3] & \rightarrow & y_3[n] = 16\delta[n-4], \\ & \vdots & \\ x_k[n] = \delta[n-k] & \rightarrow & y_k[n] = (k+1)^2\delta[n-(k+1)] \text{ for any integer k.} \end{array}$$

No

(10 pts) **a)** Can this system be time-invariant? Explain.

$x[n] \rightarrow \boxed{sys} \rightarrow (k+1)^2 \delta[\square - (k+1)] \rightarrow \boxed{TD} \rightarrow (n-N_0)$

$(k+1)^2 \delta[(n-N_0)-(k+1)]$

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$x[n] \rightarrow \boxed{TD} \rightarrow (\square - N_0) \rightarrow \boxed{sys} \rightarrow (k+1)^2 \delta[n-(k+1)]$

$((k+1)^2 \delta[n-(k+1)]) - N_0$

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?

$$x_k[n-k] = \sum_{n=0}^{\infty} \frac{\delta[n]}{(k+1)^2}$$

-10

0

$$y[n] = \sum_{k=1}^{\infty} \delta[n-k]$$

$r = k-1$

$$= \sum_{r=0}^{\infty} \delta[n-(r+1)]$$

$$= \sum_{k=0}^{\infty} \frac{y_k[n]}{(k+1)^2}$$

inverse system $\rightarrow \sum_{k=0}^{\infty} \frac{x_k[n]}{(k+1)^2}$

$$= \sum_{k=0}^{\infty} \frac{\delta[n-k]}{(k+1)^2}$$