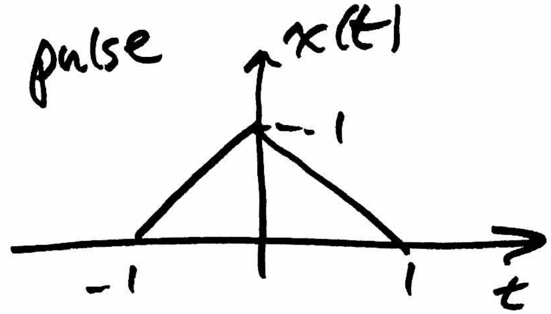
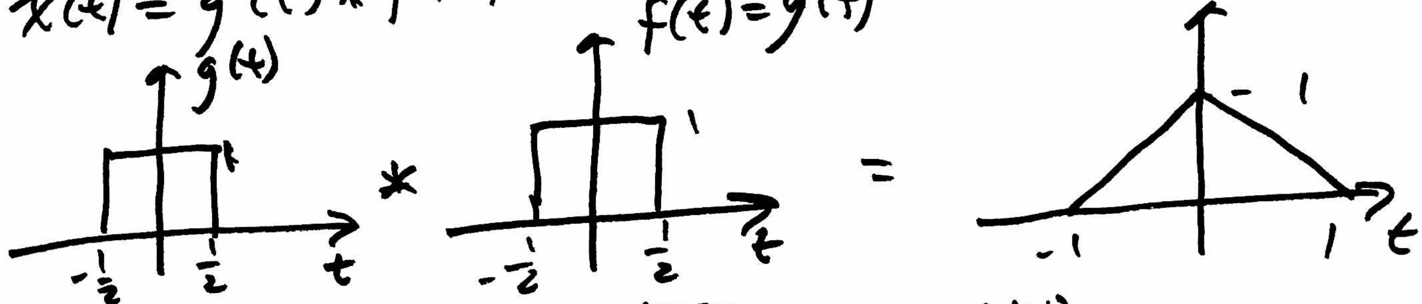
Ex Triangular pulse



- Find $X(\omega)$
  → Previously, we found ~~the~~ $x'(t)$ and used the diff. property to find $X(\omega)$.

- → We can use the convolution property instead.

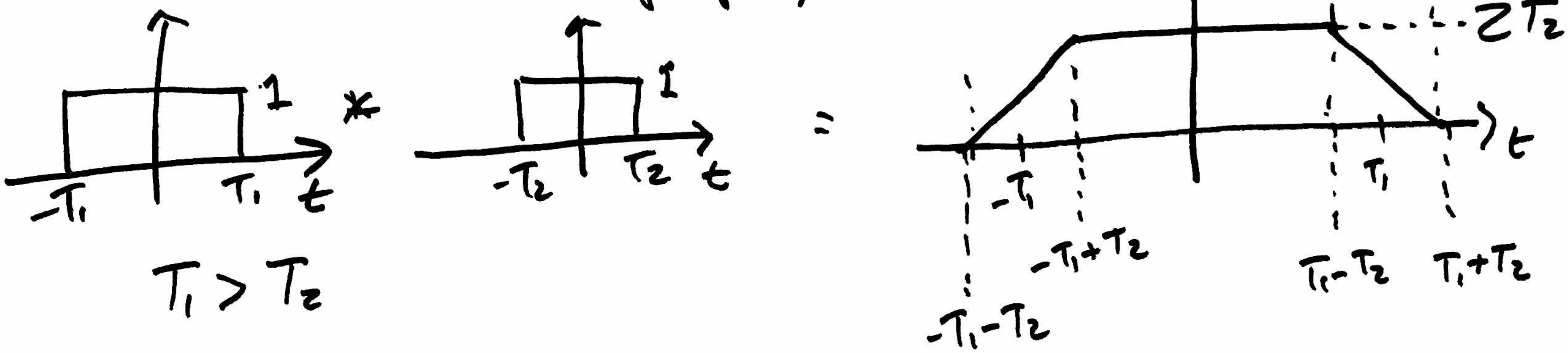
$x(t) = g(t) * f(t)$



we know that $g(t) \xleftrightarrow{\text{CTFT}} 2\,\dfrac{\sin(\frac{\omega}{2})}{\omega}$

$$\mathcal{F}\{x(t)\} = G(\omega)^2 = 4\,\frac{\sin^2(\frac{\omega}{2})}{\omega^2}$$

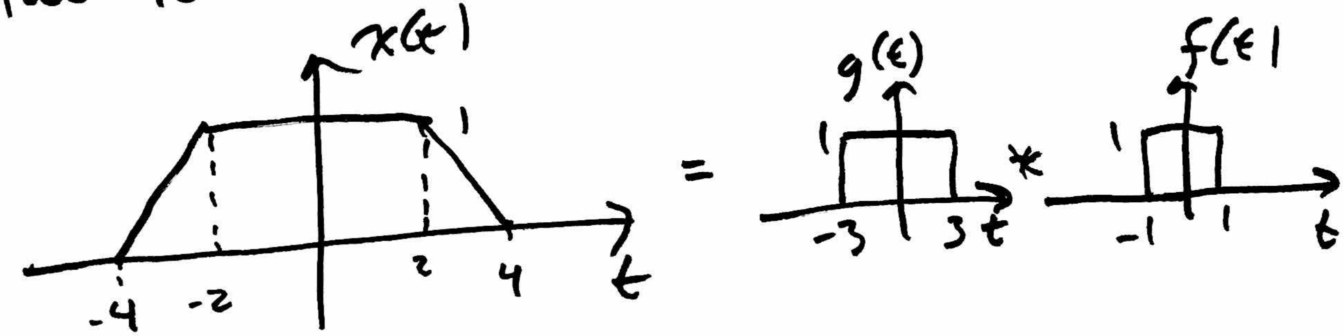
More about the convolution property



$T_1 > T_2$

If you see trapezoids, consider using the convolution of two rects.

<u>Ex</u>



Find $X(\omega)$

$X(\omega) = G(\omega)\cdot F(\omega)\cdot A \Rightarrow A = \frac{1}{2}$ so that we match

using the pair

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & \text{else} \end{cases} \longleftrightarrow 2\frac{\sin \omega T_1}{\omega}$$

$$X(\omega) = 2\frac{\sin(\omega\cdot 3)}{\omega}\cdot 2\frac{\sin(\omega\cdot 1)}{\omega}\cdot\frac{1}{2} = 2\frac{\sin(3\omega)\sin(\omega)}{\omega^2}$$

OW 4.18

$$H(\omega) = \frac{\sin^2 3\omega}{\omega^2}\cos\omega = \underbrace{\left(\frac{\sin(3\omega)}{\omega}\right)^2}_{G^2(\omega)} \cdot \underbrace{\cos\omega}_{F(\omega)}$$

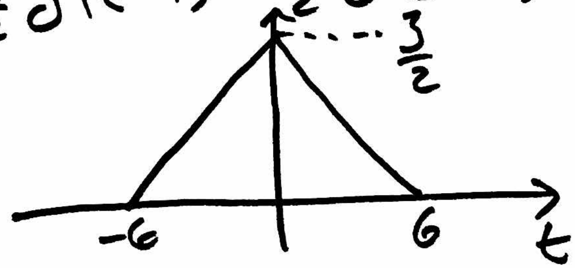
Using the table ~~G~~ $G(\omega)$ is a sinc $\rightarrow$ $g(t)$ is a rect

$$g(t) = \begin{cases} \frac{1}{2}, & |t| < 3 \\ 0, & \text{else} \end{cases}$$

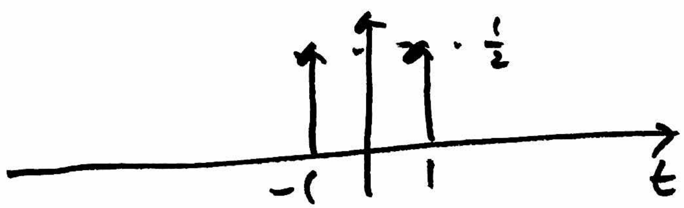
$$f(t) = \mathcal{F}^{-1}\{\cos\omega\} = \mathcal{F}^{-1}\left\{\tfrac{1}{2}e^{j\omega} + \tfrac{1}{2}e^{-j\omega}\right\}$$

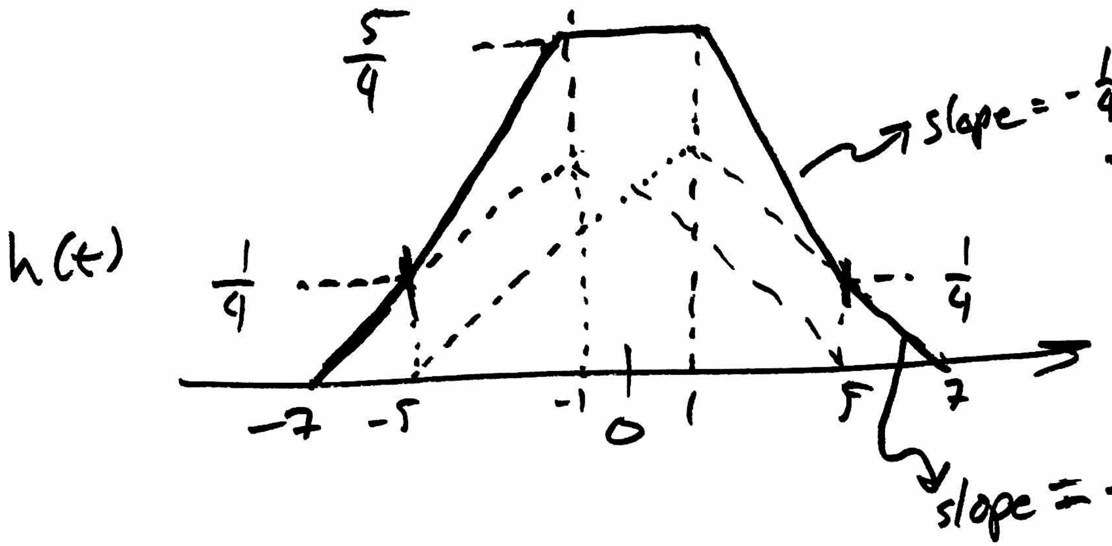
$$= \tfrac{1}{2}\delta(t-1) + \tfrac{1}{2}\delta(t+1)$$

$g(t) * g(t)$



$f(t)$





- height of each triangle is $\frac{3}{4}$
- slope of each triangle is $\pm \frac{1}{8}$