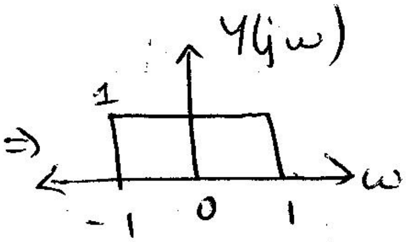
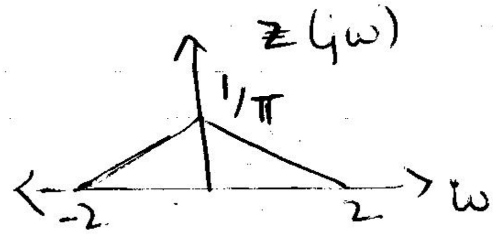
(4.10) (a) $x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$

$y(t) = \frac{\sin t}{\pi t} \xrightarrow{FT} \begin{cases} 1, & |\omega| < 1 \\ 0, & |\omega| > 1 \end{cases} \Rightarrow$



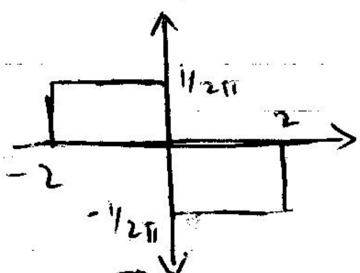
$z(t) = \frac{\sin t}{\pi t} \cdot \frac{\sin t}{\pi t} \xleftrightarrow{FT} \frac{1}{2\pi} \cdot Y(j\omega) * Y(j\omega)$

$=$

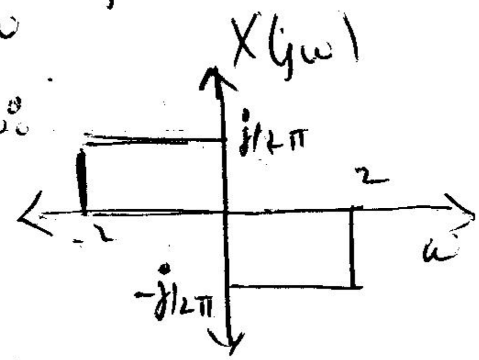


$x(t) = t\, z(t) \xrightarrow{FT} j\frac{d}{d\omega} Z(j\omega)$

$\frac{d}{d\omega} Z(j\omega) \Rightarrow$

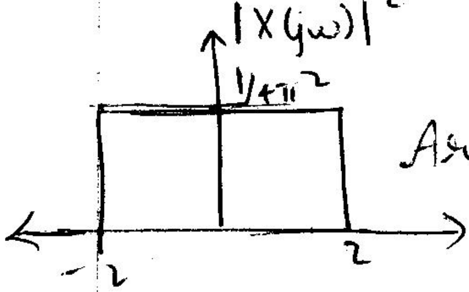


$\therefore$



(b) $A = \int_{-\infty}^{\infty} t^2 \left(\frac{\sin t}{\pi t}\right)^4 dt = \int_{-\infty}^{\infty} x(t)^2\, dt$

$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega =$



Area $= \frac{1}{4\pi^2} \cdot 4 = \frac{1}{\pi^2}$ $\therefore$ $A = \frac{1}{2\pi} \frac{1}{\pi^2} = \frac{1}{2\pi^3}$

(4.26) $x(t) = te^{-2t}u(t)$, $h(t) = e^{-4t}u(t)$

(a) (i) $H(j\omega) = \dfrac{1}{4+j\omega}$ $\quad x(t) = t\,z(t)$ $\quad \left[Z(j\omega) = \dfrac{1}{2+j\omega}\right]$

$\therefore x(t) = t\,z(t)$

$X(j\omega) = j\dfrac{d}{d\omega} Z(j\omega)$

$= j\left[-j(2+j\omega)^{-2}\right]$

$X(j\omega) = 1/(2+j\omega)^2$

$\therefore Y(j\omega) = X(j\omega)\,H(j\omega)$

$= \dfrac{1}{(2+j\omega)^2} \cdot \dfrac{1}{(4+j\omega)}$

$= \dfrac{1/4}{4+j\omega} + \dfrac{1/2}{(2+j\omega)^2} - \dfrac{1/4}{2+j\omega}$

$\therefore y(t) = \dfrac{1}{4}e^{-4t}u(t) - \dfrac{1}{4}e^{-2t}u(t) + \dfrac{1}{2}te^{-2t}u(t)$

(4.19) $\cancel{X}(j\omega) = \dfrac{Y(j\omega)}{H(j\omega)}$ $\quad H(j\omega) = 1/j\omega+3$

$Y(j\omega) = 1/j\omega+3 - 1/j\omega+4.$

$\therefore X(j\omega) = 1 - \dfrac{j\omega+3}{j\omega+4} = \dfrac{j\omega+4-j\omega-3}{j\omega+4}$

$X(j\omega) = \dfrac{1}{j\omega+4} \Rightarrow \underline{x(t) = e^{-4t}u(t)}$

4.32 $h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}$ $\quad h(t) = h_1(t-1)$

$$h_1(t) = \frac{\sin 4t}{\pi t} \xrightarrow{FT} \begin{cases} 1, & |\omega|<4 \\ 0, & |\omega|>4 \end{cases}$$

$$\therefore H(j\omega) = e^{-j\omega} H_1(j\omega) = \begin{cases} e^{-j\omega}, & |\omega|<4 \\ 0, & \text{otherwise} \end{cases}$$

(a) $x_1(t) = \cos(6t + \pi/2)$

$$X_1(j\omega) = e^{j\omega\pi/12}\, \pi\left[\delta(\omega-6) + \delta(\omega+6)\right]$$

$$= e^{-j\pi/2} + e^{j\pi/2} = 2\cos\pi/2 = \underline{\underline{0}}.$$

$$\therefore Y_1(j\omega) = 0 \quad \therefore y_1(t) = 0$$

(b) $x_2(t) = \sum_0^{\infty} \left(\frac{1}{2}\right)^k \sin(3kt)$

$$X_2(j\omega) = \frac{\pi}{j}\left[\sum_0^{\infty}\left(\frac{1}{2}\right)^k \left\{\delta(\omega-3k) - \delta(\omega+3k)\right\}\right]$$

$$Y_2(j\omega) = X_2(j\omega)H(j\omega) = \frac{\pi}{j}\left[\sum_0^{\infty}\left(\frac{1}{2}\right)^k \left\{\delta(\omega-3k) - \delta(\omega+3k)\right\}\right]e^{-j\omega}$$

$$= \frac{\pi}{j}\left[\frac{1}{2}\left(\delta(\omega-3) - \delta(\omega+3)\right)e^{-j\omega}\right] \quad \left[\begin{matrix}\text{Only} \\ \text{at } k=1\end{matrix}\right]$$

$$= \frac{\pi}{2\pi}\left[\frac{e^{3j\omega} - e^{-3j\omega}}{2j}\right]$$

$$= \frac{1}{2}\sin(3t-1)$$

(c) $x_3(t) = \dfrac{\sin(4(t+1))}{\pi(t+1)}$

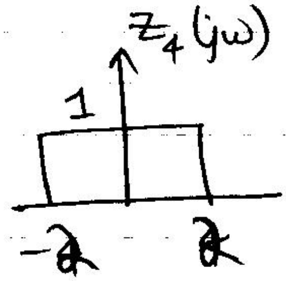
$$X_3(j\omega) = \begin{cases} e^{j\omega}, & |\omega| < 4 \\ 0, & |\omega| > 4. \end{cases}$$

$$Y_3(j\omega) = X_3(j\omega)H(j\omega) = X_3(j\omega)e^{-j\omega}$$

$$\Rightarrow y_3(t) = x_3(t-1) = \frac{\sin(4t)}{\pi t}$$

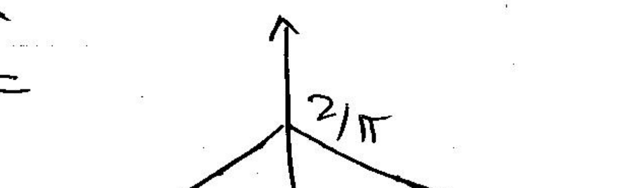
(d) $x_4(t) = \left(\dfrac{\sin 2t}{\pi t}\right)^2 = \underbrace{\dfrac{\sin 2t}{\pi t}}_{z_4(t).} \cdot \dfrac{\sin 2t}{\pi t}$

$Z_4(j\omega) =$



$\therefore X_4(j\omega) = \dfrac{1}{2\pi} Z_4 * Z_4$

$\Rightarrow X_4(j\omega) =$



$$\therefore Y_4(j\omega) = X_4(j\omega)H(j\omega) = X_4(j\omega)e^{-j\omega}$$

$$y(t) = \left(\frac{\sin(2(t-1))}{\pi(t-1)}\right)^2$$

(4.33) (a) $H(j\omega) = \dfrac{2}{(j\omega)^2 + 6(j\omega) + 8} = \dfrac{2}{(j\omega+2)(j\omega+4)}$

$$= \frac{1}{j\omega+2} - \frac{1}{j\omega+4}.$$

$$h(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

(b) $y(t) = x(t) * h(t) \xrightarrow{F.T.} Y(j\omega) = X(j\omega)H(j\omega)$

$$x(t) = te^{-2t}u(t) \qquad X(j\omega) = \frac{1}{(2+j\omega)^2}$$

$$Y(j\omega) = \frac{2}{(j\omega+2)^3(j\omega+4)} = \frac{A}{(j\omega+2)} + \frac{B}{(j\omega+2)^2} + \frac{C}{(j\omega+2)^3} + \frac{D}{j\omega+4}$$

$$= \frac{1/4}{j\omega+2} - \frac{1/2}{(j\omega+2)^2} + \frac{1}{(j\omega+2)^3} - \frac{1/4}{j\omega+4}$$

$$y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + t^2e^{-2t}u(t) - 1/4\, e^{-4t}u(t)$$

(c) $H(j\omega) = \dfrac{2(j\omega)^2 - 2}{(j\omega)^2 + \sqrt{2}j\omega + 1}$

$$= 2 + \frac{-\sqrt{2} - 2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}+j\sqrt{2}}{2}} + \frac{-\sqrt{2} + 2\sqrt{2}j}{j\omega - \frac{-\sqrt{2}-j\sqrt{2}}{2}}$$

$$h(t) = 2\delta(t) - \sqrt{2}(1+2j)e^{-(1+j)t/\sqrt{2}}u(t) - \sqrt{2}(1-2j)e^{-(1-j)t/\sqrt{2}}u(t)$$

(4.34) (a) $\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 4x(t)$

(b) $H(j\omega) = \frac{j\omega+4}{(j\omega)^2+5j\omega+6} = \frac{j\omega+4}{(j\omega+3)(j\omega+2)}$

$$= \frac{2}{2+j\omega} - \frac{1}{3+j\omega}$$

$$h(t) = 2e^{-2t}u(t) - e^{-3t}u(t)$$

(c) $X(j\omega) = \frac{1}{j\omega+4} - \frac{1}{(j\omega+4)^2}$

$$Y(j\omega) = X(j\omega)H(j\omega) = \frac{1}{(4+j\omega)(2+j\omega)}$$

$$= -\frac{1}{2}\frac{1}{4+j\omega} + \frac{1}{2}\frac{1}{2+j\omega}$$

$$y(t) = \frac{1}{2}e^{-2t}u(t) - \frac{1}{2}e^{-4t}u(t)$$

(4.45) $y(t) = x(t) * h(t) \qquad Y(j\omega) = X(j\omega)H(j\omega)$

$$E = \int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 |H(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_0-\Delta/2}^{-\omega_0+\Delta/2} |X(j\omega)|^2 d\omega + \frac{1}{2\pi}\int_{\omega_0-\Delta/2}^{\omega_0+\Delta/2} |X(j\omega)|^2 d\omega$$

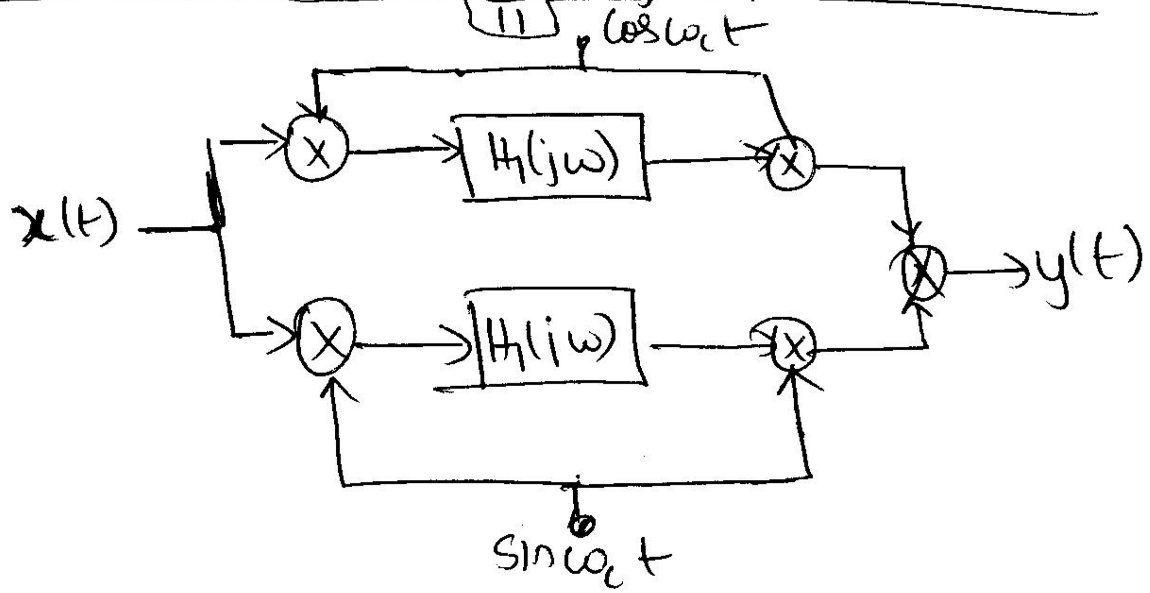
But his is area at $\pm\omega_0 \approx X(j\pm\omega_0)\cdot\Delta$

$$\therefore \approx \frac{1}{2\pi}|X(-j\omega_0)|^2\Delta + \frac{1}{2\pi}|X(j\omega_0)|^2\Delta$$

Real $x(t)$, $|X(-j\omega_0)|^2 = |X(j\omega_0)|^2$

$$\therefore E \approx \boxed{\frac{1}{\pi}}|X(j\omega_0)|^2\Delta$$

(4.46)



~~See past~~ Let $X_1(t)$ be for $\cos\omega_c t$
$X_2(t)$ be for $\sin\omega_c t$.