Homework 6

8.1) $X(j\omega) = 0 \quad \text{for } |\omega| > \omega_m$

$$Y(j\omega) = 2X\{j(\omega - \omega_c)\}$$

$$y(t) = 2x(t)e^{j\omega_c t}$$

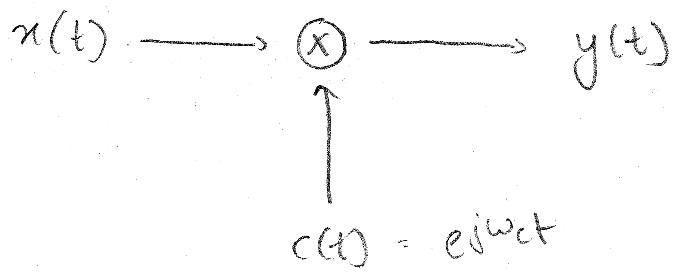
$$y(t) = x(t)m(t)$$

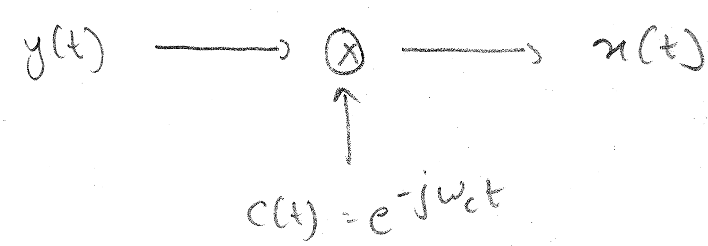
$$\therefore \; m(t) = 2e^{j\omega_c t}$$

8.2 (a) $X(j\omega) = 0 \quad \text{when } |\omega| > 1000\pi$

$$y(t) = e^{j\omega_c t}x(t)$$

i.e. $c(t) = e^{j\omega_c t}$



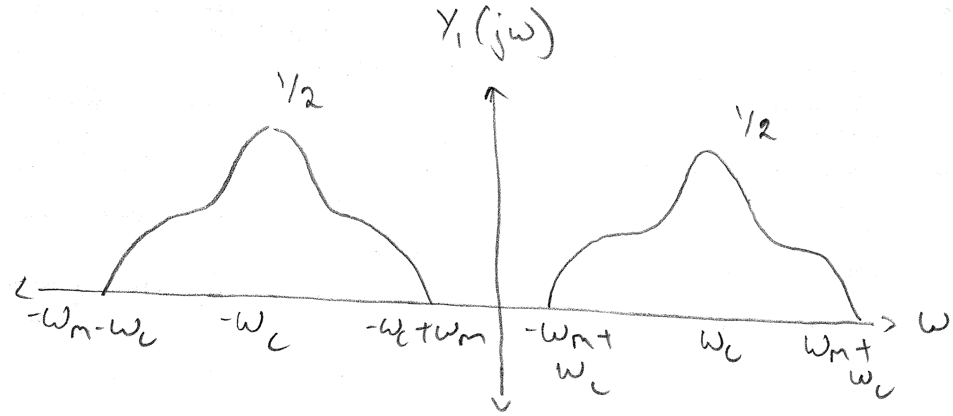


$\therefore$ No constraint on ω_c is required to ensure that $x(t)$ is recoverable from $y(t)$.

(b) $$y(t) = e^{j\omega_c t}\, x(t)$$
$$= [\cos(\omega_c t) + j\sin(\omega_c t)]\, x(t)$$

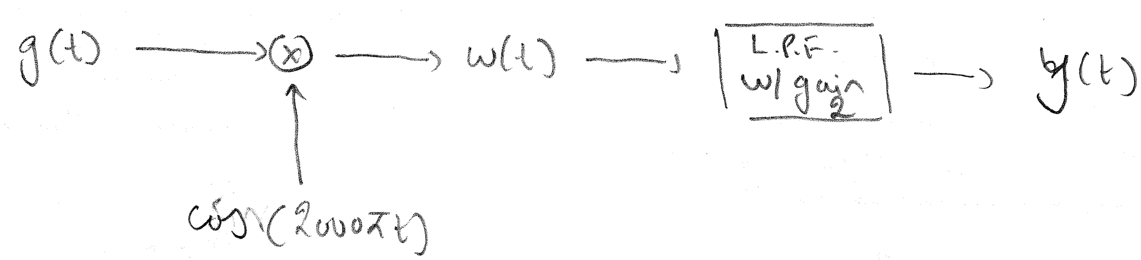
$$Re\{y(t)\} = \cos(\omega_c t)\, x(t) = y_1(t)$$

$$Y_1(j\omega) = \frac{1}{2}\left[X\{j(\omega-\omega_c) + X\{j(\omega+\omega_c)\}\right]$$



∴ <u>Constraint.</u> $$-\omega_m + \omega_c > -\omega_c + \omega_m$$
$$2\omega_c > 2\omega_m$$
$$\omega_c > \omega_m$$
$$|\omega_c| > 1000\pi.$$ Ans.

8.3) $$g(t) = x(t)\sin(2000\pi t)$$



$$w(t) = g(t) \overset{\cos}{\cancel{\sin}} (2000\pi t)$$

$$= x(t) \sin(2000\pi t) \cos(2000\pi t)$$

$$= \frac{1}{2} x(t) \sin(4000\pi t)$$

$$w(t) \longrightarrow \boxed{\begin{matrix}\text{L.P.F. w/} \\ \text{gain } 2\end{matrix}} \longrightarrow y(t) = 0$$

$\boxed{\text{OR}}$

$$W(j\omega) = \frac{1}{2\pi} \cdot \frac{1}{2} X(j\omega) * \frac{\pi}{j}\left[X\{j(\omega - 4000\pi) + X\{j + 4000\pi\}\right.$$

$$= \frac{1}{4j} \; X\left\{ j(\omega - 4000\pi)\right\} + X\left\{ j(\omega + 4000\pi)\right\}$$

$X(j\omega) = 0$ for $|\omega| > 2000\pi$ {given}

$\therefore$ $y(t) = 0$ after passing through a L.P.F. w/ gain 2.

5.1) (a) $x[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{n-1} u[n-1]\, e^{-j\omega n}$$

$$= \sum_{n=1}^{\infty} (1/2)^{n-1}\, e^{-j\omega n}$$

$m = n-1$

$$= \sum_{m=0}^{\infty} (1/2)^{m}\, e^{-j\omega(m+1)}$$

$$= e^{-j\omega} \sum_{m=0}^{\infty} (1/2)^{m}\, e^{-j\omega m}$$

$$= e^{-j\omega} \cdot \frac{1}{(1 - 1/2\, e^{-j\omega})}$$

5.2 (a) $\delta[n-1] + \delta[n+1] = x[n]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left\{ \delta[n-1] + \delta[n+1] \right\} e^{-j\omega n}$$

$$= e^{-j\omega n}\Big|_{n=-1} + e^{-j\omega n}\Big|_{n=1}$$

$$= e^{j\omega} + e^{-j\omega}$$

$$= 2\cos(\omega).$$ Ans

$5.3(a) = \sin\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$ { periodic signal }

$$N = \frac{2\pi}{\omega} = \frac{2\cancel{\pi}}{\cancel{\pi}/3} = 6$$

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta\left(\omega - \frac{2\pi}{N}k\right)$$

$$\therefore \cancel{X(e^{j\omega})} : \quad x[n] = \frac{1}{2j} e^{j(\pi/3 n + \pi/4)} - \frac{1}{2j} e^{-j(\pi/3 n + \pi/4)}$$

$$\therefore \quad a_1 = \frac{1}{2j} e^{j\pi/4}$$

$$a_0 = a_2 = a_3 = a_4 = 0$$

$$a_{-1} = -\frac{1}{2j} e^{-j\pi/4} = a_5$$

$$\therefore \quad X(e^{j\omega}) = \cancel{2}\pi \frac{1}{\cancel{2}j} e^{j\pi/4} \delta(\omega - \pi/3) +$$

$$\cancel{2}\pi\left(-\frac{1}{\cancel{2}j}\right) e^{-j\pi/4} \delta(\omega - 2\pi/6)$$

$$= \frac{\pi}{j}\left[e^{j\pi/4}\delta(\omega - \pi/3) - e^{-j\pi/4}\delta(\omega - \pi/3)\right]$$

5.4.(a) $X_1(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \{ 2\pi\delta(\omega - 2\pi k) + \pi\delta(\omega - \pi/2 - 2\pi k)$

$+ \pi\delta(\omega + \pi/2 - 2\pi k) \}$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X_1(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} [2\pi\delta(\omega) + \pi\delta(\omega - \pi/2) + \pi\delta(\omega + \pi/2)]\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{\cancel{2\pi}}\cdot\cancel{2\pi}\, e^{j(0)n} + \frac{1}{2\cancel{\pi}}\cancel{\pi}\, e^{j(\pi/2)n} + \frac{1}{2\cancel{\pi}}\cancel{\pi}\, e^{j(-\pi/2)n}$$

$$= 1 + \tfrac{1}{2} e^{j\pi/2 n} + \tfrac{1}{2} e^{-j\pi/2 n}$$

$$= 1 + \cos(\pi/2\, n).$$

5.5) (a) $$|X(e^{j\omega})| = \begin{cases} 1 & 0 \le |\omega| < \pi/4 \\ 0 & \pi/4 \le |\omega| \le \pi \end{cases}$$

$$\measuredangle X(e^{j\omega}) = -3/2\,\omega$$

$$X(e^{j\omega}) = |X e^{j\omega}|\, e^{j\measuredangle X(e^{j\omega})}$$

$$\therefore\ x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X e^{j\omega}|\, e^{j\measuredangle X(e^{j\omega})}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} (1)\, e^{-j\,3/2\,\omega}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j\omega(n-3/2)}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(n-3/2)}}{j(n-3/2)}\Bigg|_{-\pi/4}^{\pi/4}\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\pi/4(n-3/2)}}{j\,\cancel{\pi/4}\,(n-3/2)} - \frac{e^{-j\pi/4(n-3/2)}}{j\,\cancel{\pi/4}\,(n-3/2)}\right]$$

$$= \frac{\sin\left[\pi/4\,(n-3/2)\right]}{\pi(n-3/2)}$$

$$x[n] = 0 = \frac{\sin\left[\pi/4\,(n-3/2)\right]}{\pi(n-3/2)}$$

$\therefore\ n = \pm\infty$ for $x[n] = 0$. Ans.

5.6)(a) $x_1[n] = x[1-n] + x[-1-n]$

$$= x[-n+1] + x[-n-1]$$

$$X_1(e^{j\omega}) = e^{j\omega} X(e^{-j\omega}) + e^{-j\omega} X(e^{-j\omega})$$

$$= X(e^{-j\omega})(e^{j\omega} + e^{-j\omega})$$

$$= 2X(e^{-j\omega})\cos(\omega).$$

5.7)(a) $X_1(e^{j\omega}) = e^{-j\omega} \sum_{k=1}^{10} \sin(k\omega)$

$$Y_1(e^{j\omega}) = \sum_{k=1}^{10} \sin(k\omega)$$

$\therefore$ $Y_1(e^{j\omega})$ is real & odd

$$X_1(e^{j\omega}) = e^{-j\omega(1)} Y_1(e^{j\omega})$$

$$x_1[n] = y_1[n-1]$$

(i) Imaginary

(ii) Neither odd nor even.

5.8) $X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}\left(\frac{\sin 3/2\omega}{\sin \omega/2}\right) + 5\pi\delta(\omega)$

5.9) $\Im m\{X(e^{j\omega})\} = \sin\omega - \sin 2\omega$

$$j\,\Im m\{X(e^{j\omega})\} = j\sin\omega - j\sin 2\omega$$

$$Od\{x[n]\} \xrightarrow{F.T.} j\,\Im m\{X(e^{j\omega})\}$$

$$Od\{x[n]\} = j\sin\omega - j\sin 2\omega$$

$$= \frac{1}{2}\left[e^{j\omega} - e^{-j\omega} - e^{j2\omega} + e^{-j2\omega}\right]$$

$$x_o[n] = \quad = \frac{1}{2}\left[\delta[n+1] - \delta[n-1] - \delta[n+2] + \delta[n-2]\right] \longrightarrow (1)$$

$$Od\{x[n]\} = \frac{x[n] - x[-n]}{2}$$

$$2\,Od\{x[n]\} = x[n] - x[-n]$$

i.e. $x[n] = \delta[n+1] - \delta[n+2]$ $\quad \{ x[n] = 0$ for $n > 0$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2\, d\omega = 3 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

i.e. $3 = \sum_{n=-\infty}^{-1} |x[n]|^2 + |x[0]|^2$

$$= 2 + |x[0]|^2$$

$$x[0] = \pm 1$$

$$\therefore\ x[0] = 1 \qquad \{ x[0] > 0 \}$$

$\therefore \quad x[n] = x[0] + \delta[n+1] - \delta[n-2]$

$= \delta[n] + \delta[n-1] - \delta[n-2]$. Ans

5.19) $\quad y[n] - \frac{1}{6} y[n-1] - \frac{1}{6} y[n-2] = x[n]$

$$Y(e^{j\omega}) \left[1 - \frac{1}{6} e^{-j\omega} - \frac{1}{6} e^{-j\omega 2} \right] = X(e^{j\omega})$$

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = H(e^{j\omega}) = \frac{1}{1 - \frac{1}{6} e^{-j\omega} - \frac{1}{6} e^{-j\omega 2}}$$

$$= \frac{1}{(1 - \frac{1}{2} e^{-j\omega})(1 + \frac{1}{3} e^{-j\omega})}$$

$$\therefore \quad \overset{H(e^{j\omega})}{\cancel{h[n]}} = \frac{A}{1 - \frac{1}{2} e^{-j\omega}} + \frac{B}{1 + \frac{1}{3} e^{-j\omega}}$$

$$= \frac{3/5}{1 - 1/2\, e^{-j\omega}} + \frac{2/5}{1 + 1/3\, e^{-j\omega}}$$

$$h[n] = 3/5 \, (1/2)^n u[n] + 2/5 \, (-1/3)^n u[n].$$

5.20) $\left(\frac{4}{5}\right)^n u[n] \longrightarrow n\left(\frac{4}{5}\right)^n u[n]$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\mathcal{F}\left[n\left(\frac{4}{5}\right)^n u[n]\right]}{\mathcal{F}\left[\left(\frac{4}{5}\right)^n u[n]\right]}$$

$$= \frac{j\,\frac{d}{d\omega} X(e^{j\omega})}{X(e^{j\omega})}$$

$$= \frac{\dfrac{\frac{4}{5}e^{-j\omega}}{(1-\frac{4}{5}e^{-j\omega})^2}}{\dfrac{1}{1-\frac{4}{5}e^{-j\omega}}}$$

$$= \frac{\frac{4}{5}e^{-j\omega}}{1-\frac{4}{5}e^{-j\omega}}$$

$$Y(e^{j\omega})\left(1-\frac{4}{5}e^{-j\omega}\right) = X(e^{j\omega})\,\frac{4}{5}e^{-j\omega}$$

$$y[n] - \frac{4}{5}y[n-1] = \frac{4}{5}x[n-1]$$ Ans:

5.21) (a) $x[n] = u[n-2] - u[n-6]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} [u[n-2] - u[n-6]] e^{-j\omega n}$$

$$= \sum_{n=2}^{5} e^{-j\omega n}$$

$$= e^{-j\omega 2} + e^{-j\omega 3} + e^{-j\omega 4} + e^{-j\omega 5}.$$ Ans

$$x[n] = \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5].$$ Ans

(c) $x[n] = (\frac{1}{3})^{|n|} u[-n-2]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{j\omega n}$$

$$= \sum_{n=-\infty}^{-1} (1/3)^{-n} u[-n-2] e^{-j\omega n} + \sum_{n=1}^{\infty} (\tfrac{1}{3})^{n} u[-n-2] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-2} (1/3)^{-n} e^{-j\omega n} + \cancel{\sum_{n=2}^{\infty} (1/3)^{n} e^{-j\omega n}} \to 0$$

$$= \sum_{n=2}^{\infty} (1/3)^{n} e^{j\omega n}$$

$$= \sum_{n=2}^{\infty} [(1/3)(e^{j\omega})]^{n}$$

$$m = n - 2$$

$$X(e^{j\omega}) = \sum_{m=0}^{\infty} \left(\tfrac{1}{3} e^{j\omega}\right)^{m+2}$$

$$= \sum_{m=0}^{\infty} \left(\tfrac{1}{3} e^{j\omega}\right)^{m} \left(\tfrac{1}{3} e^{j\omega}\right)^{+2}$$

$$= \frac{e^{j\omega 2}}{9} \sum_{n=0}^{\infty} \left(\tfrac{1}{3} e^{j\omega}\right)^{n}$$

$$= \frac{e^{j\omega 2}}{9} \cdot \frac{1}{1 - \tfrac{1}{3} e^{j\omega}}.$$ Ans.

(g) $x[n] = \sin\left(\tfrac{\pi}{2} n\right) + \cos(n)$

$$X(e^{j\omega}) = \frac{\pi}{j}\left[\delta\left(\omega - \tfrac{\pi}{2}\right) - \delta\left(\omega + \tfrac{\pi}{2}\right)\right] + \pi\left[\delta(\omega - 1) + \delta(\omega + 1)\right]$$

$\cos(n)$ is not periodic

$\sin\left(\tfrac{\pi}{2} n\right) \longleftrightarrow N = \cancel{\text{to}}\ \frac{2\pi}{\omega} = \frac{2\cancel{\pi}}{\cancel{\pi}/2} = 4$

(i) $x[n] = x[n-6]$

$x[n] = u[n] - u[n-5]$

$N = 6$

$$a_k = \frac{1}{N} \sum_{n=0}^{5} x[n]\, e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{6} \sum_{n=0}^{\cancel{5}4} (1)\, e^{-j\frac{\pi}{3}kn}$$

$$= \frac{1}{6}\left[\frac{1-e^{-j\frac{5\pi}{3}k}}{1-e^{-j(\frac{\pi}{3})k}}\right]$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi\, a_k\, \delta\left(\omega - \frac{2\pi}{N}k\right)$$

$\boxed{OR}$

$$X(e^{j\omega}) = \sum_{l=-\infty}^{\infty} 2\pi\, a_k\, \delta\left(\omega - \frac{2\pi}{N} - 2\pi l\right)$$

$$= \sum_{l=-\infty}^{\infty} 2\pi\left(\frac{1}{6}\right)\left[\frac{1-e^{-j\frac{5\pi}{3}k}}{1-e^{-j(\frac{\pi}{3})k}}\right]\delta\left(\omega - \frac{2\pi}{6} - 2\pi l\right.$$

(k) $$x[n] = \left[\frac{\sin(\pi n \cancel{8}/5)}{\pi n}\right]\cos\left(\frac{7\pi}{2}n\right)$$

$$= x_1[n] \cdot x_2[n]$$

$$X_1(e^{j\omega}) = \begin{cases} 1 & 0 \le |\omega| < \pi/5 \\ 0 & \pi/5 < |\omega| < \pi \end{cases}$$

w/ $N = 10$ & periodic w/ 2π.

$$X_2(e^{j\omega}) = \pi \sum_{k=-\infty}^{\infty} \left\{ \delta\left(\omega - \frac{7\pi}{2} - 2\pi k\right) + \delta\left(\omega + 7\pi/2 - 2\pi k\right) \right\}$$

$$\therefore\; X(e^{j\omega}) = \frac{1}{2\pi} X_1(e^{j\omega}) * X_2(e^{j\omega})$$

$$= \frac{1}{2\cancel{\pi}} X_1(e^{j\omega}) * \cancel{\pi} \left\{ \delta(\omega - 7\pi/2) + \delta(\omega + 7\pi/2) \right\}$$

$$= \frac{1}{2}\left[X_1\{e^{j(\omega - 7\pi/2)}\} + X_1\{e^{j(\omega + 7\pi/2)}\} \right]$$

$$= \frac{1}{2}\left[X_1\{e^{j(\omega - \pi/2)}\} + X_1\{e^{j(\omega + \pi/2)}\} \right]$$

$$\therefore\; X(e^{j\omega}) = \begin{cases} 1/2 & 3\pi/10 < |\omega| < 7\pi/10 \\ 0 & \text{otherwise.} \end{cases}$$

5.24) $\mathrm{Re}\{X(e^{j\omega})\} = 0$

i.e. the signal is real & odd

$\therefore$ the transform is purely imaginary and odd.

$\therefore$ (b) & (d)

5.29) (a) $h[n] = (1/2)^n u[n]$

(i) $x[n] = (3/4)^n u[n]$

$$X(e^{j\omega}) = \frac{1}{1 - 3/4\, e^{-j\omega}}.$$

$$H(e^{j\omega}) = \frac{1}{1 - 1/2\, e^{-j\omega}}$$

$$Y(e^{j\omega}) = \frac{1}{(1 - 3/4\, e^{-j\omega})} \cdot \frac{1}{(1 - 1/2\, e^{-j\omega})}$$

$$= \frac{-2}{1 - 3/4\, e^{-j\omega}} + \frac{3}{1 - 1/2\, e^{-j\omega}}$$

$$y[n] = 3(3/4)^n u[n] \cancel{+} - 2(1/2)^n u[n].$$

5.33) (a) $y[n] + \frac{1}{2} y[n-1] = x[n]$

$$Y(e^{jw}) \left[1 + \frac{1}{2} e^{-jw} \right] = X(e^{jw})$$

$$H(e^{jw}) = \frac{1}{1 + \frac{1}{2} e^{-jw}}$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

(b) (i) $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$$X(e^{jw}) = \frac{1}{1 - \frac{1}{2} e^{-jw}}$$

$$Y(e^{jw}) = \left(\frac{1}{1 + \frac{1}{2} e^{-jw}}\right) \left(\frac{1}{1 - \frac{1}{2} e^{-jw}}\right)$$

$$= \frac{1/2}{1 + \frac{1}{2} e^{-jw}} + \frac{1/2}{1 - \frac{1}{2} e^{-jw}}$$

$$= \frac{1}{2}(1/2)^n u[n] + 1/2\,(-1/2)^n u[n].$$

(c) (i) $X(e^{jw}) = \dfrac{1 - \frac{1}{4} e^{-jw}}{1 + \frac{1}{2} e^{-jw}}$

$$Y(e^{j\omega}) = \left[\frac{1-\frac{1}{4}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}}\right] \cdot \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

$$= \frac{1}{(1+\frac{1}{2}e^{-j\omega})^2} + \frac{\frac{1}{4}e^{-j\omega}}{(1+\frac{1}{2}e^{-j\omega})^2}$$

$$= (n+1)\left(-\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)\delta[n-1] \cdot (n+1)\left(-\frac{1}{2}\right)^n u[n]$$

$$= (n+1)\left(-\frac{1}{2}\right)^n u[n] + \frac{1}{4}\, n\left(-\frac{1}{2}\right)^{n-1} u[n-1].$$

Ans.