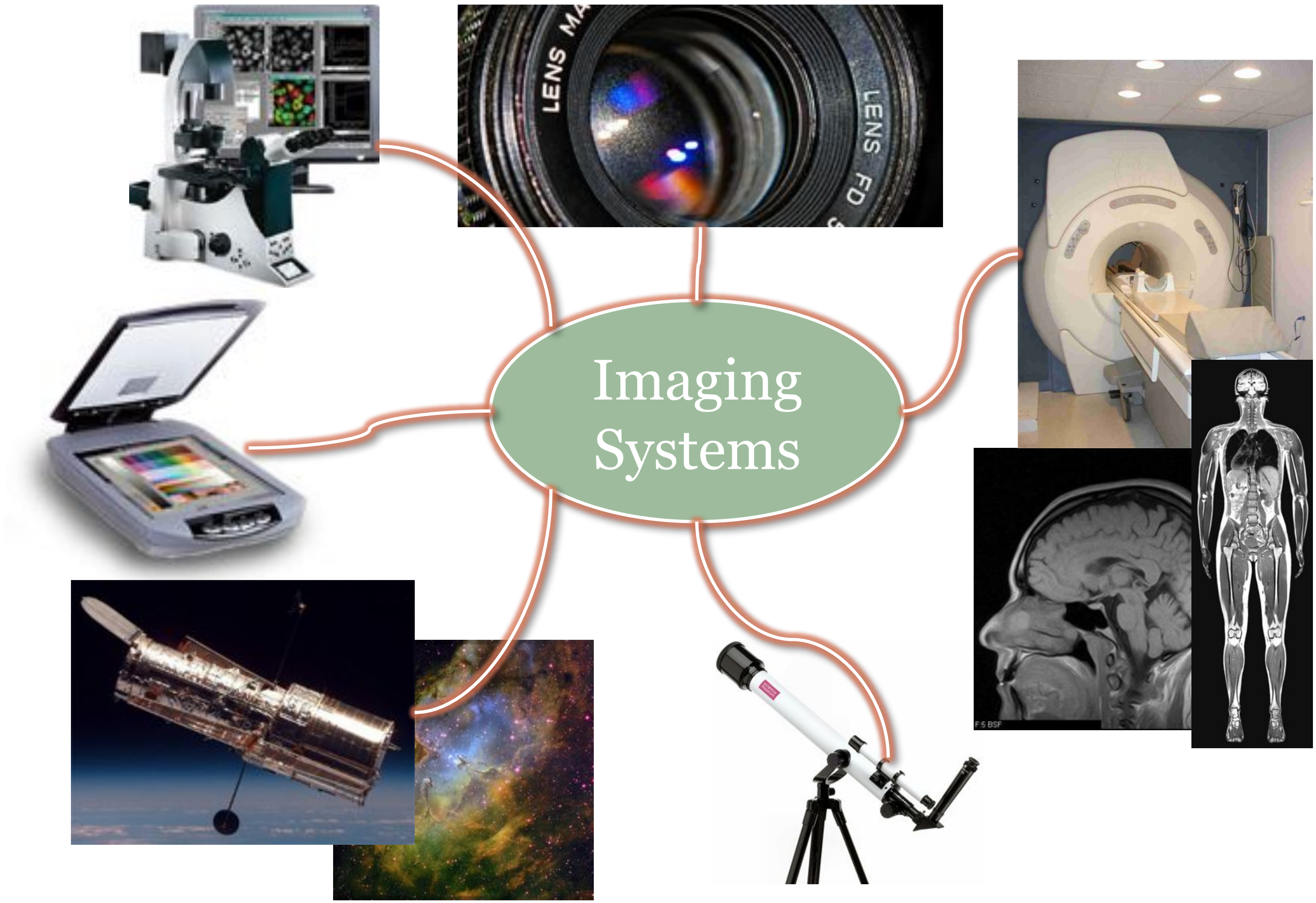
# Linear, Shift Invariant Imaging Systems



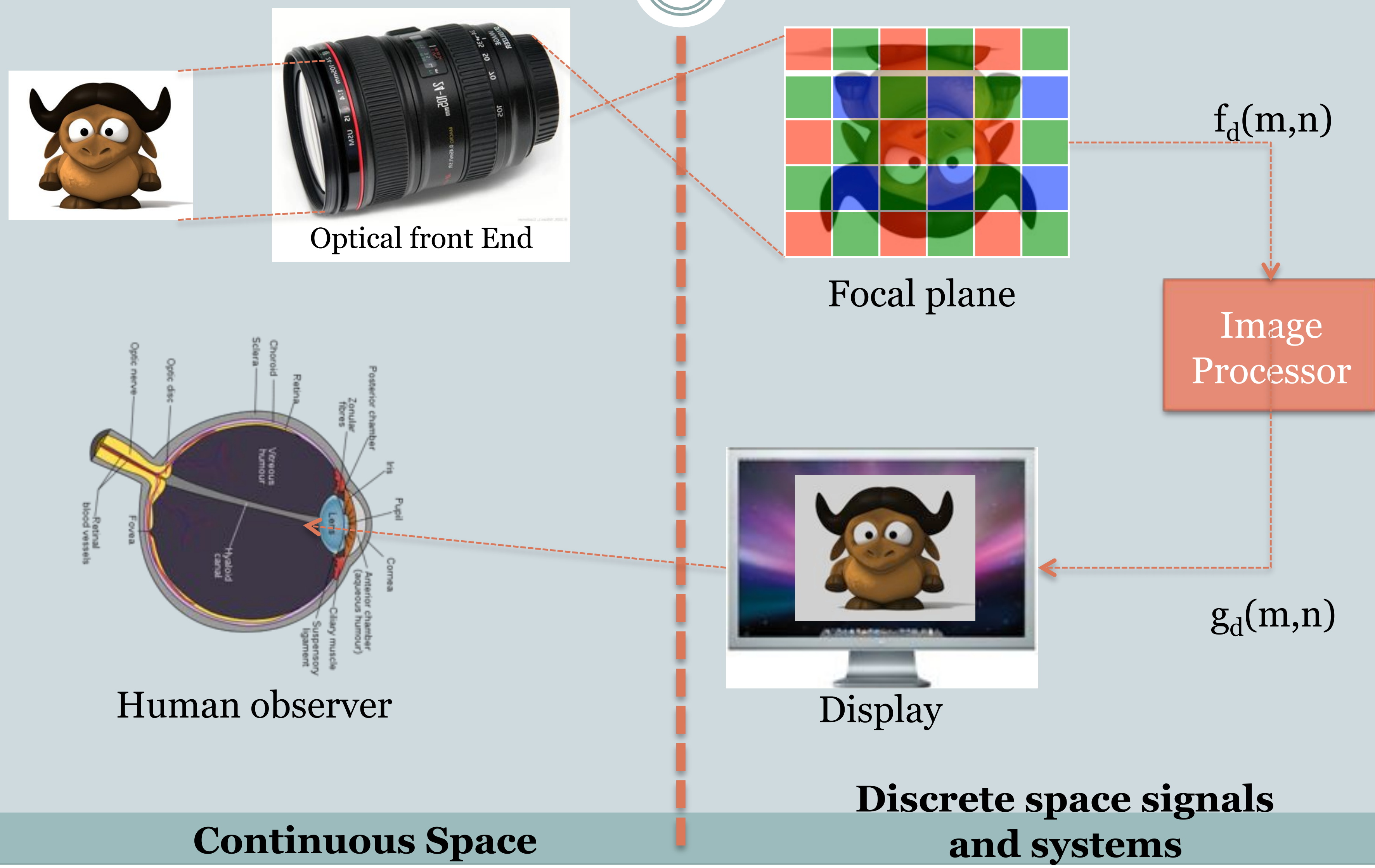
MONDAY, NOVEMBER 11TH, 2009

HECTOR SANTOS-VILLALOBOS




*We will concentrate on digital incoherent systems such as digital cameras.

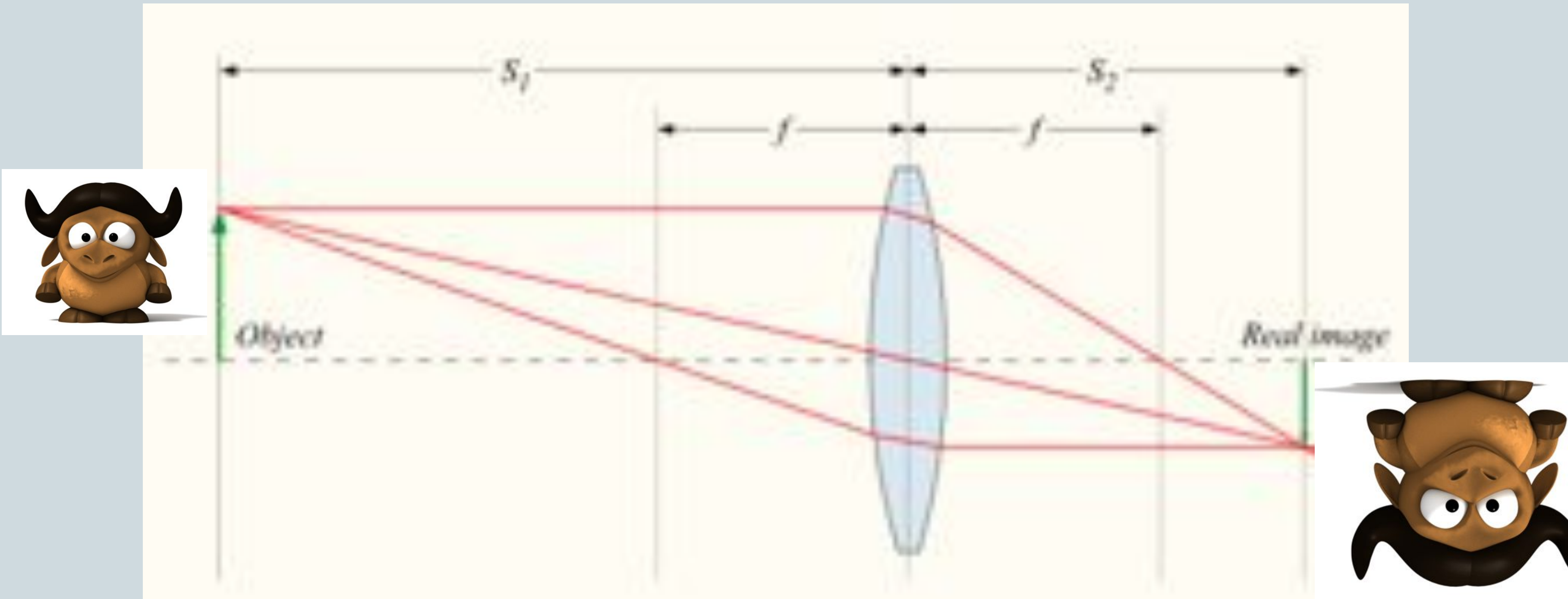
# Overview


Optical front End
Focal plane
$f_d(m,n)$
Image Processor
$g_d(m,n)$
Display
Optic nerve
Optic disc
Sclera
Choroid
Retina
Posterior chamber
Zonular fibres
Iris
Pupil
Cornea
Lens
Vitreous humour
Hyaloid canal
Fovea
Retinal blood vessels
Anterior chamber (aqueous humour)
Ciliary muscle
Suspensory ligament
Human observer
Continuous Space
Discrete space signals and systems

# How does a camera work?



*neglecting thickness of lens




Focal length:

$$\frac{1}{f} = \frac{1}{S_1} + \frac{1}{S_2}$$

Magnification:

$$M = \frac{f}{f - S_1} = -\frac{S_2}{S_1}$$

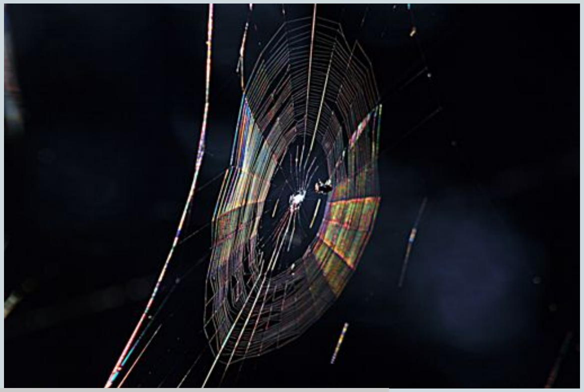
# Aberrations



$$\delta(x,y) \rightarrow \underbrace{h(x,y)}_{} \neq \delta(x,y)$$

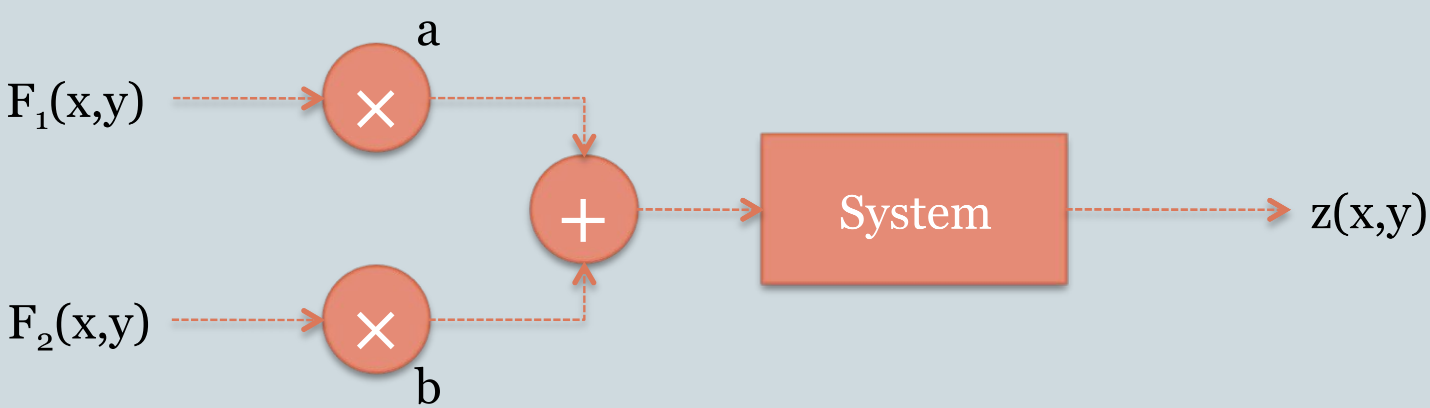
Point spread function

- Diffraction
  - Caused by aperture
  - Creates a "airy disk" pattern
- Lens geometry
  - Edges of lens create problems
  - Shape imperfections
- Dispersion
  - Refractive index of light depends on its wavelength

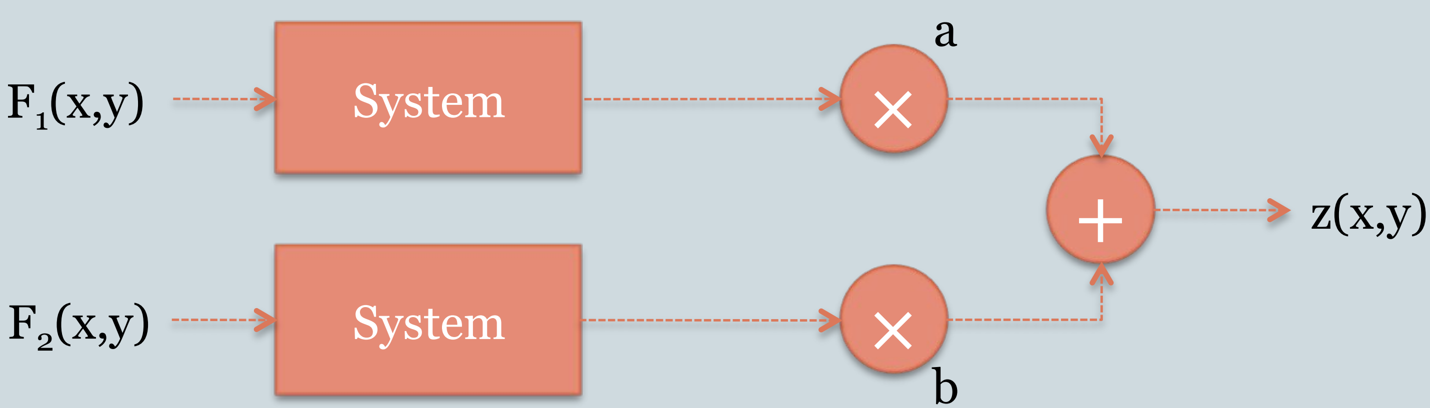


# Linear continuous-space system





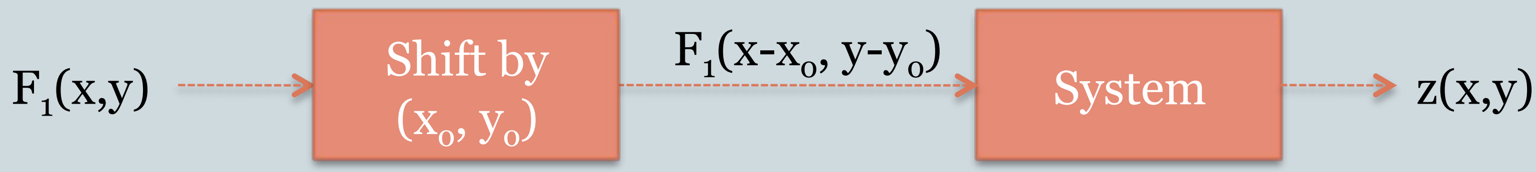

**Same as:**




*Same for discrete-space systems

# Shift invariant continuous-space system






**Same as:**


*Same for discrete-space systems

# Note on shift invariance



In our camera system, we do not have shift invariance. Let h(x,y) be the system's response to δ(x,y)

$$\delta(x,y) \rightarrow h(x,y)$$

Then,

$$\delta(x-\epsilon, y-\eta) \not\rightarrow h(x-\epsilon, y-\eta)$$

*Because of magnification factor.

Rather,

$$\delta(x-\epsilon, y-\eta) \not\rightarrow h(x-M\epsilon, y-M\eta)$$

$$M = \frac{S_2}{S_1}$$

In order to assume shift invariance need to assume magnification M=1.

# Note on linearity



**Claim:** Output of linear, shift invariant imaging systems can be computed by convolving the input with the point spread function.
Because,

$$f(x, y) = \iint_{\Re^2} f(u, v)\delta(x - u, y - v)dudv$$

Then,

$$\begin{aligned}\text{image } I(x, y) &= \iint_{\Re^2} f(u, v)\ [image\, of\ \delta(x - u, y - v)]\, dudv \\ &= \iint_{\Re^2} f(u, v)h(x - u, y - v)dudv \\ &= f(x, y) \star h(x, y)\end{aligned}$$