Linear, Shift Invariant Imaging Systems

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*We will concentrate on digital incoherent systems such as digital cameras.*
Overview

Optical front End

Focal plane

Image Processor

Display

Human observer

Continuous Space

Discrete space signals and systems

\[ f_d(m,n) \]

\[ g_d(m,n) \]
How does a camera work?

Focal length:
\[
\frac{1}{f} = \frac{1}{S_1} + \frac{1}{S_2}
\]

Magnification:
\[
M = \frac{f}{f - S_1} = -\frac{S_2}{S_1}
\]

*neglecting thickness of lens
Aberrations

\[ \delta(x, y) \rightarrow h(x, y) \neq \delta(x, y) \]

- **Diffraction**
  - Caused by aperture
  - Creates a “airy disk” pattern

- **Lens geometry**
  - Edges of lens create problems
  - Shape imperfections

- **Dispersion**
  - Refractive index of light depends on its wavelength

Point spread function
Linear continuous-space system

\[ F_1(x,y) \times a \]
\[ F_2(x,y) \times b \]

System

\[ z(x,y) \]

Same as:

\[ F_1(x,y) \text{ System} \]
\[ a \times + \]

\[ F_2(x,y) \text{ System} \]
\[ b \times + \]

\[ z(x,y) \]

*Same for discrete-space systems*
Shift invariant continuous-space system

 Same as:

*Same for discrete-space systems
Note on shift invariance

In our camera system, we do not have shift invariance. Let $h(x,y)$ be the system’s response to $\delta(x,y)$

$$\delta(x, y) \rightarrow h(x, y)$$

Then,

$$\delta(x - \epsilon, y - \eta) \not\rightarrow h(x - \epsilon, y - \eta)$$

*Because of magnification factor.

Rather,

$$\delta(x - \epsilon, y - \eta) \not\rightarrow h(x - M\epsilon, y - M\eta)$$

$$M = \frac{S_2}{S_1}$$

In order to assume shift invariance need to assume magnification $M=1$. 
**Claim:** Output of linear, shift invariant imaging systems can be computed by convolving the input with the point spread function. Because,

\[ f(x, y) = \int_{\mathbb{R}^2} f(u, v) \delta(x - u, y - v) dudv \]

Then,

\[ \text{image } I(x, y) = \int_{\mathbb{R}^2} f(u, v) [\text{image of } \delta(x - u, y - v)] dudv \]

\[ = \int_{\mathbb{R}^2} f(u, v) h(x - u, y - v) dudv \]

\[ = f(x, y) \ast h(x, y) \]