1. For each function given below, do the following:
i. Sketch $f(x, y)$
ii. Express $f(x, y)$ in terms of the special functions given in class.
iii. Find its CSFT $F(u, v)$ using transform pairs and properties.
iv. Sketch $F(u, v)$ in enough detail to show that you know what it looks like.
a. $\quad f(x, y)=\left\{\begin{array}{cc}\cos (2 \pi(x-y)), & x^{2}+y^{2}<4 \\ 0, & \text { else }\end{array}\right.$
b.

2. The 2-D signal $f(x, y)=1+\cos (2 \pi(3 x+y))$ is sampled with an ideal sampler at 4 samples/inch to generate the signal

$$
f_{s}(x, y)=\sum_{m} \sum_{n} f(0.25 m, 0.25 n) \delta(x-0.25 m, y-0.25 n)
$$

This signal is then convolved with $\operatorname{sinc}(4 \mathrm{x}, 4 \mathrm{y})$ to yield the reconstructed signal $\mathrm{f}_{\mathrm{r}}(x, y)$.
a. Sketch $\mathrm{f}(x, y)$ showing a top view of the $x-y$ plane in which the points where $\mathrm{f}(x, y)=1$ are clearly labeled.
b. Find a simple expression for $\mathrm{f}_{\mathrm{r}}(x, y)$.
c. Sketch $\mathrm{f}_{\mathrm{r}}(x, y)$ showing a top view of the $x-y$ plane in which the points where $\mathrm{f}_{\mathrm{r}}(x, y)=1$ are clearly labeled.
3. Consider a $3 \times 3$ FIR filter with coefficients $h[m, n]$

| m |  |  |  |
| :--- | :--- | :--- | :--- |
| n | -1 | 0 | 1 |
| 1 | -0.125 | 0.5 | -0.125 |
| 0 | -0.25 | 1.0 | -0.25 |
| -1 | -0.125 | 0.5 | -0.125 |

a. Find a difference equation that can be used to implement this filter.
b. Find the output image that results when this filter is applied to the input image shown below:

| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

c. Find a simple expression for the frequency response (DSFT) $H(\mu, v)$ of this filter.
d. Plot $H(\mu, v)$ along the $\mu$ axis $(v=0)$, along the $v$ axis $(\mu=0)$, along the line $\mu=v$, and along the line $\mu=-v$.
e. Discuss the relation between your answer to part b. and the filter frequency response.

