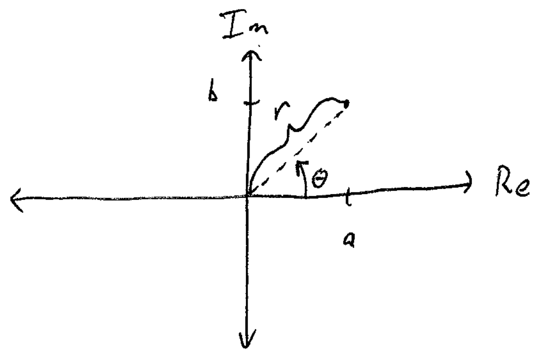
Complex Numbers

there are two ways to think about complex numbers:

rectangular coordinates : $a + jb$ $\qquad j = \sqrt{-1}$

polar coordinates : $re^{j\theta}$



$a = r\cos\theta \qquad b = r\sin\theta$

$r = \sqrt{a^2+b^2}$

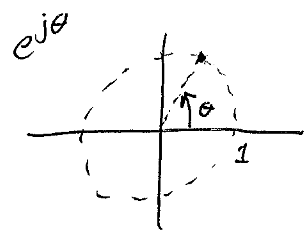
$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Euler's Identity

$e^{j\theta} = \cos\theta + j\sin\theta$

$\Rightarrow \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta} = \frac{1}{2}\cos\theta + \frac{1}{2}j\sin\theta + \frac{1}{2}\cos\theta - \frac{1}{2}j\sin\theta$

$= \cos\theta$



$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$

$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$

$\mathrm{Re}\{e^{j\theta}\} = \cos\theta$ "real part"

$\mathrm{Im}\{e^{j\theta}\} = \sin\theta$ "imaginary part"

Note : $\mathrm{Im}\{z\}$ should not have a j in it!

$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$

Remember : This is an angle on the unit circle $\Rightarrow$ radius (magnitude) is 1!

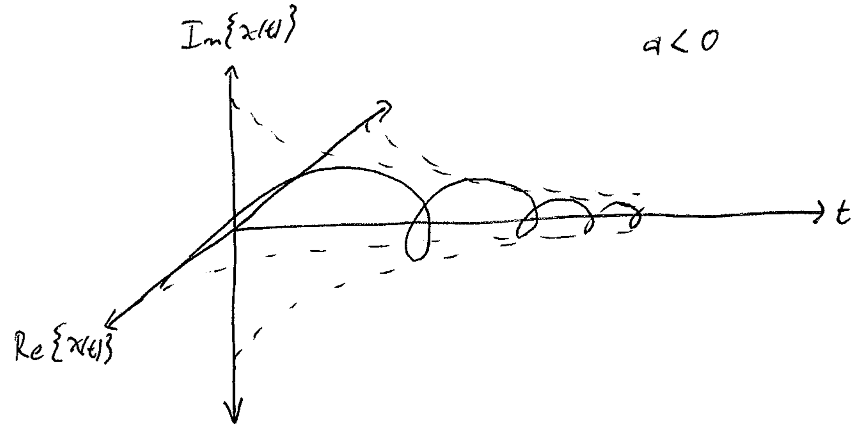
Complex Exponentials

Let $x(t) = e^{(a+jb)t}$

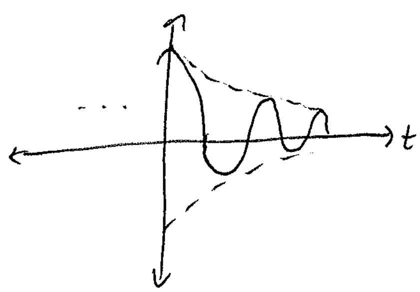
$= e^{at} e^{jbt}$

$|x(t)| = |e^{at}||e^{jbt}| = e^{at}$

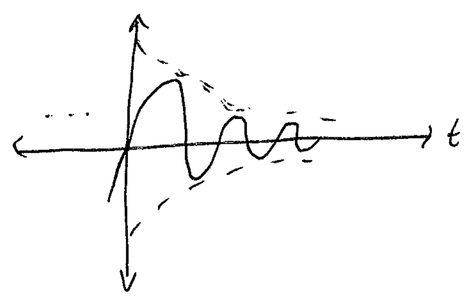
$\angle x(t) = bt$



$Re\{x(t)\} = e^{at}\cos(bt)$



$Im\{x(t)\} = e^{at}\sin(bt)$



Periodicity:

Since $e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$ & sin and cos are periodic,

$e^{j\omega t}$ is also periodic.

$$e^{j\omega(t+T)} = e^{j\omega T} e^{j\omega t}$$

$$e^{j\omega T} = 1 \Rightarrow \omega T = 2\pi k \qquad k \in \mathbb{Z}$$

$$T = \frac{2\pi}{\omega}$$