Signal Properties

Energy

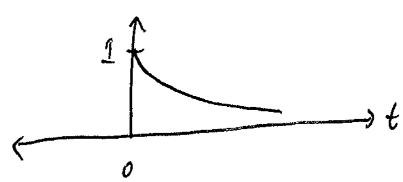
$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt \qquad \text{OR} \qquad E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2$$

Remember the power through a resistor was $i^2(t)R$ or $\frac{v^2(t)}{R}$

and therefore the energy was $\int i^2(t) R \, dt$ or $\int \frac{v^2(t)}{R} dt$.

Here we are doing the same thing but neglecting units.

Ex: $x(t) = e^{-\alpha t} u(t) \qquad \alpha > 0$



$$E_\infty = \lim_{T\to\infty} \int_0^T e^{-2\alpha t} dt$$

$$= \lim_{T\to\infty} \frac{-1}{2\alpha} e^{-2\alpha t} \Big|_0^T$$

$$= \lim_{T\to\infty} \frac{-1}{2\alpha}\left(e^{-2\alpha T} - 1\right) = \frac{1}{2\alpha}$$

finite energy

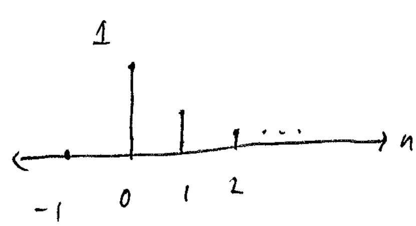
/Ex

Ex $x(t) = 1$

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} (1)\, dt = \lim_{T\to\infty} t \Big|_{-T}^{T} = \lim_{T\to\infty} 2T = \infty$$

/Ex infinite energy

Ex $x[n] = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & \text{ow} \end{cases}$



$$E_\infty = \lim_{N\to\infty} \sum_{n=0}^{N} \left(\tfrac{1}{2}\right)^{2n}$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} \left(\tfrac{1}{4}\right)^{n}$$

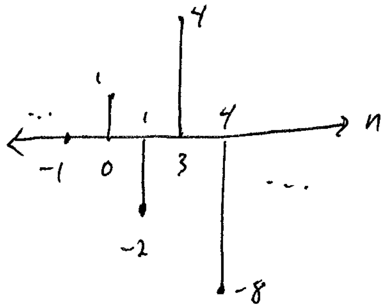
$$= \lim_{N\to\infty} \frac{1 - \left(\frac{1}{4}\right)^{N+1}}{1 - \frac{1}{4}} = \frac{1}{1 - \frac{1}{4}}$$

$$= \frac{4}{3}$$

finite energy

/Ex

Ex: $x[n] = \begin{cases} (-2)^n & n \geq 0 \\ 0 & ow \end{cases}$



$$E_\infty = \lim_{N \to \infty} \sum_{n=0}^{N} (2)^{2n} = \lim_{N \to \infty} \sum_{n=0}^{N} 4^n$$

$$= \lim_{N \to \infty} \frac{1 - 4^{N+1}}{1-4} = \infty$$

infinite energy

/Ex

Power

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt \quad \text{OR} \quad P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Note: $P_\infty = \lim_{T \to \infty} \frac{E_\infty}{2T}$ So, finite energy $\Rightarrow P_\infty = 0$

also, finite power $\Rightarrow E_\infty = \infty$

Ex: $x(t) = \cos(\omega t) \quad \omega > 0$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \cos^2(\omega t) \, dt$$

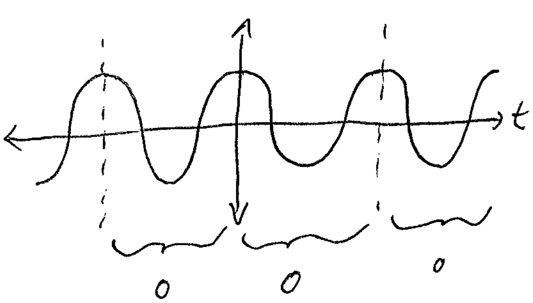
$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{2}\left(1 + \cos(2\omega t)\right) dt$$

$$= \lim_{T \to \infty} \left[\frac{1}{4T} t \Big|_{-T}^{T} + \frac{1}{4T} \int_{-T}^{T} \cos(2\omega t) \, dt \right]$$

$$= \lim_{T \to \infty} \left[\frac{1}{4T}(2T) + \frac{1}{4T}\frac{1}{2\omega} \sin(2\omega t)\Big|_{-T}^{T} \right]$$

$$= \frac{1}{2} + 0 \quad \text{finite power}$$

Think about integrating $\cos(2\omega t)$ this way:



Each period integrates to zero. So, adding up the integrals over several periods is still zero.

/Ex

Ex: $x[n] = (-1)^n$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (1)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} (2N+1) = 1 \quad \text{finite power}$$

Note: $\sum_{n=-N}^{N} (1) = \underbrace{1+1+\cdots+1}_{-N \text{ to } -1} + \underbrace{1}_{n=0} + \underbrace{1+\cdots+1}_{1 \text{ to } N}$

$N \quad 1 \quad N \longrightarrow 2N+1$

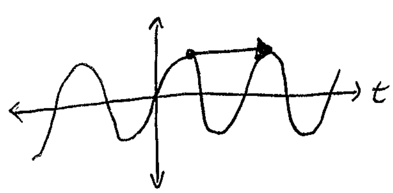
/Ex

<u>Periodicity</u>

If a signal can be shifted such that every point still lines up,

$x(t+T) = x(t)$ for all $t \in \mathbb{R}$, then it is said to be "periodic."

Ex: $x(t) = \sin(4t)$ sin & cos are periodic with period 2π



$x(t+T) = \sin(4t+4T)$

if $4T = 2\pi k$ $\quad k \in \mathbb{Z}$

then $x(t+T) = \sin(4t) = x(t)$

So, $x(t)$ is period with period $T = \frac{\pi}{2}$.

/Ex

Ex $x(t) = t^2$

$x(t+T) = (t+T)^2 = t^2 + 2Tt + T^2$

there is no value of T other than zero that makes

$x(t+T) = x(t)$. Therefore, $x(t)$ is aperiodic.

/Ex

Ex: $x[n] = \cos(n)$

$x[n+N] = \cos(n+N)$

If $N = 2\pi k$, $k \in \mathbb{Z}$ then $x[n+N] = x[n]$

But BE CAREFUL! N has to be an <u>integer</u>

There is no integer value of N that makes $x[n+N] = x[n]$

N can't be an integer <u>and</u> a multiple of 2π.

/Ex So, $x[n]$ is aperiodic.

Ex $x[n] = (-1)^n$

$x[n+N) = (-1)^{n+N}$

$= (-1)^N (-1)^n$

If $N = 2k$, $k \in \mathbb{Z}$ then $x[n+N] = (-1)^n = x[n]$

/Ex Therefore, $x[n]$ is periodic with period 2.

<u>Even</u>

If $x(-t) = x(t)$, then the signal is called "even".

Ex: $x(t) = |t|$

$x(-t) = |-t| = |t| = x(t)$

/Ex Therefore, $x(t)$ is even.

Ex: $x[n] = \cos(n)$ Note: cos is even

$x[-n] = \cos(-n) = \cos(n) = x[n]$

/Ex Therefore, $x[n]$ is even.

Odd

If $x(-t) = -x(t)$, then the signal is "odd".

Ex: $x(t) = \sin(\omega t)$ $\omega > 0$

$x(-t) = \sin(-\omega t) = -\sin(\omega t)$ Note: sin is odd

$= -x(t)$

/Ex therefore, $x(t)$ is odd.

Ex $x[n] = n^3$

$x[-n] = (-n)^3 = -n^3 = -x[n]$

/Ex therefore, $x[n]$ is odd.