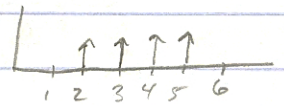
Lecture 19
10/15/08 TA Lecture

Quiz - show that FT of $x(t) = e^{j2\pi t}$

is $X(\omega) = 2\pi\,\delta(\omega - 2\pi)$

__Examples__

$x[n] = u[n-2] - u[n-6]$ in DT $\rightarrow$



find $X(\omega)$

rewrite $x[n] = \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$

we know $\mathcal{F}(\delta[n-N]) = e^{-Nj\omega}$

$\therefore \quad X(\omega) = e^{-2j\omega} + e^{-3j\omega} + e^{-4j\omega} + e^{-5j\omega}$

ex/ $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n]\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{e^{-j\omega}}{2}\right)^n$$

geometric series $\Rightarrow \dfrac{1}{1 - \frac{e^{-j\omega}}{2}}$

ex/

$$x[n] = \left(\frac{1}{4}\right)^{|n|} = \begin{cases} \left(\frac{1}{4}\right)^{n} & n \geq 0 \\ \left(\frac{1}{4}\right)^{-n} & n < 0 \end{cases}$$

$$= \left(\frac{1}{4}\right)^{n} u[n] + \left(\frac{1}{4}\right)^{-n} u[-n-1]$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^{n} u[n] e^{-j\omega n} + \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^{-n} u[-n-1] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{e^{-j\omega}}{4}\right)^{n} + \sum_{n=-\infty}^{-1} \left(\frac{1}{4}\right)^{-n} e^{-j\omega n}$$

$$= \frac{1}{1 - \frac{e^{-j\omega}}{4}} + \sum_{m=1}^{\infty} \left(\frac{1}{4}\right)^{m} e^{j\omega m} \qquad m = -n$$

$$'' \quad + \sum_{m=1}^{\infty} \left(\frac{e^{j\omega}}{4}\right)^{m} \qquad \text{geometric series}$$

$$= \frac{1}{1 - \frac{e^{-j\omega}}{4}} + \left(\frac{1}{1 - \frac{e^{j\omega}}{4}} - 1\right)$$

ex/ $x[n] = \sin\left(\frac{\pi}{2} n\right) u[n]$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \sin(\pi/2\, n) u[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \sin\left(\frac{\pi}{2} n\right) e^{-j\omega n} = \frac{1}{2j} \sum_{n=0}^{\infty} \left[e^{j\pi/2\, n} - e^{-j\pi/2\, n} \right] e^{-j\omega n}$$

$$= \frac{1}{2j} \left[\sum_{n=0}^{\infty} e^{j\pi/2\, n} e^{-j\omega n} - \sum_{n=0}^{\infty} e^{-j\pi/2\, n} e^{-j\omega n} \right]$$

$$= \frac{1}{2j} \left[\sum_{n=0}^{\infty} \left(\frac{e^{j\pi/2}}{e^{j\omega}} \right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{e^{j\pi/2 + j\omega}} \right)^n \right]$$

$$= \frac{1}{2j} \left[\frac{1}{1 - e^{-j(\omega - \pi/2)}} - \frac{1}{1 - e^{-j(\omega + \pi/2)}} \right]$$

ex/

$$X(\omega) = 1 + 3e^{-j\omega} + 2e^{-2j\omega} - 4e^{-3j\omega} + e^{-j10\omega}$$

recognize $\mathcal{F}^{-1}\{e^{-jn_0\omega}\} = \frac{1}{2\pi}\delta[n-n_0]$

$$x[n] = \frac{1}{2\pi}\left[\delta[n] + 3\delta[n-1] + 2\delta[n-2] - 4\delta[n-3] + \delta[n-10]\right]$$

ex/

$$H(\omega) = \frac{\sin^2(3\omega)\cos(\omega)}{\omega^2}$$

$$= \frac{\sin(3\omega)}{\omega} \cdot \frac{\sin(3\omega)}{\omega} \cdot \cos(\omega)$$

$$h[n] = F^{-1}\left\{\frac{\sin(3\omega)}{\omega}\right\} * F^{-1}\left\{\frac{\sin(3\omega)}{\omega}\right\} * \underbrace{F^{-1}\{\cos(\omega)\}}_{\delta(\omega-1)+\delta(\omega+1)}$$

use the table