Midterm Examination 2
ECE 301
Division 3, Fall 2007

Instructor: Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the mean-time, read the instructions below and fill out the requested info.

2. You have 50 minutes to complete the 5 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.

3. This booklet contains 10 pages. The last four pages contain a table of formulas and properties. You may tear out these pages **once the exam begins**. TABLE USE RULES: You may use any fact contained in the table without justification. Simply write the number of the corresponding table item to indicate which fact you are using from the table. If you use a non-trivial fact that is *not* contained in the table, you must justify (i.e., prove) it in order to get full credit.

4. This is a closed book exam. Calculators, cell phones, and i-pods are strictly forbidden.

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Itemized Scores

Problem 1:	11/15
Problem 2:	3/20
Problem 3:	5/15
Problem 4:	10/10
Problem 5:	10/10
Total:	39/70

Fail < 37

37 ≤ C ≤ 49

50 ≤ A, B ≤ 70

38 ← → 39

1 6 2 4 11 6 8 16 10 3 9 3 1 3

0-4 5-9 10 15 20 25 30 35 40 45 50 55 60 65 70

11

(15 pts) **1.** Using the definition of the Fourier transform (*not* the table of Fourier transform pairs), compute the Fourier transform of the DT signal:

$$x[n] = \left(\frac{1}{2j}\right)^{|n|}.$$

$$\mathcal{F}(x[n]) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \qquad (25)$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2j}\right)^{|n|} e^{-j\omega n} = \sum_{n=-\infty}^{-1} \left(\frac{1}{2j}\right)^{-n} e^{-j\omega n} + \sum_{n=0}^{\infty} \left(\frac{1}{2j}\right)^{n} e^{-j\omega n}$$

$$= \quad m = n+1 \qquad n = m-1$$

$$\frac{1}{1-\frac{1}{2j}e^{-j\omega}}$$

$$\sum_{m=-\infty+1}^{0} \left(\frac{1}{2j}\right)^{-m+1} e^{-j\omega m + j\omega}$$

$$= \frac{e^{j\omega}}{2j} \sum_{m=-\infty}^{0} \left(\frac{1}{2j}\right)^{-m} e^{-j\omega m} = \frac{e^{j\omega}}{2j\left(1-\frac{1}{2j}e^{-j\omega}\right)}$$

NO

$$\boxed{X(\omega) = \frac{e^{j\omega}}{2j\left(1-\frac{1}{2j}e^{-j\omega}\right)} + \frac{1}{1-\frac{1}{2j}e^{-j\omega}}}$$

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(20 pts) **2.** The Frequency response of a continuous-time LTI system is

$$H(j\omega) = \mathcal{H}(\omega) = \frac{1}{j\omega + 2}.$$

Use the convolution property of the Fourier transform to determine the response $y(t)$ when the input is $x(t) = e^{-|t|}$.

$y(t) = x(t) * h(t) \xrightarrow{\mathcal{F}} \mathcal{Y}(\omega) = \mathcal{X}(\omega)\mathcal{H}(\omega)$, (15)

Now, use this!

~~$x(t) = e^{-|t|} \xrightarrow{\mathcal{F}} \int_{-\infty}^{\infty} e^{-|t|} e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{t} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-t} e^{-j\omega t}\, dt$~~

~~$= e$~~ *not working for me* ~~$e^{t(-1-j\omega)}$~~

use table.

$h(t) = e^{-2t} u(t)$ by (7)

$y(t) = h(t) * x(t)$

$= \int_{-\infty}^{\infty} h(t-\tau)\, x(\tau)\, d\tau = \int_{-\infty}^{\infty} e^{-2(t-\tau)} u(t-\tau)\, e^{-|\tau|}\, d\tau$

$= e^{-2t} \int_{-\infty}^{t} e^{2\tau} e^{-|\tau|}\, d\tau = e^{-2t} \int_{-\infty}^{0} e^{2\tau} e^{\tau}\, d\tau + e^{-2t} \int_{0}^{t} e^{2\tau} e^{-\tau}$

$= e^{-2t} \left(\int_{-\infty}^{0} e^{3\tau}\, d\tau + \int_{0}^{t} e^{\tau}\, d\tau \right)$

$$\boxed{= e^{-2t} \left(\frac{1}{3} e^{3\tau} \Big|_{-\infty}^{0} + e^{\tau} \Big|_{0}^{t} \right)}$$

5

(15 pts) **3.** True/False? The Fourier transform of a DT signal $x[n]$ is a periodic function, no matter what $x[n]$ is. (Justify your answer.)

True.

by (25)

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

~~which is a sum of shifted impulses (deltas) which will always give a periodic function.~~

also by (26)

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} d\omega$$

says that $x[n]$ can be found by integrating over 1 period of 2π of $X(\omega)$ which means that $X(\omega)$ must be periodic w/ 2π.

$\cancel{9}\ 10$ / 10

(10 pts) **4.** A continuous-time LTI system has frequency response

$$H(j\omega) = \frac{j\omega + 4}{(j\omega+2)(j\omega+3)}.$$

Derive a differential equation representing this system. (Use the properties of the Fourier transform listed in the table to justify your answer.)

$$Y(\omega) = H(\omega)X(\omega) \Rightarrow \frac{Y(\omega)}{X(\omega)} = H(\omega) = \frac{j\omega+4}{(j\omega+2)(j\omega+3)}$$

$$\Leftrightarrow (j\omega+2)(j\omega+3)\,Y(\omega) = X(\omega)\,(j\omega+4)$$

$$\Leftrightarrow \left((j\omega)^2 + 5j\omega + 6\right) Y(\omega) = X(\omega)(j\omega+4)$$

$$\Leftrightarrow (j\omega)^2 Y(\omega) + 5j\omega\, Y(\omega) + 6Y(\omega) = j\omega X(\omega) + 4X(\omega) \quad \text{, by (9)}$$

$$\boxed{\Leftrightarrow \frac{d y^2(t)}{dt} + 5\frac{d y(t)}{dt} + 6y(t) = \frac{d x(t)}{dt} + 4x(t)} \quad \text{, by (16)}$$

Don't write "=" when you mean "$\Leftarrow$".

10

(10 pts) **5.** A CT signal $x(t)$ has Fourier transform

$$\mathcal{X}(\omega) = -2e^{(j-1)\omega}u(\omega+1).$$

Denote by $y(t)$ the signal obtained by delaying $x(t)$ by six seconds. Sketch a graph representing the magnitude $|\mathcal{Y}(\omega)|$ of the Fourier transform $\mathcal{Y}(\omega)$ of $y(t)$. (Justify your answer.)

$y(t) = x(t-6)$

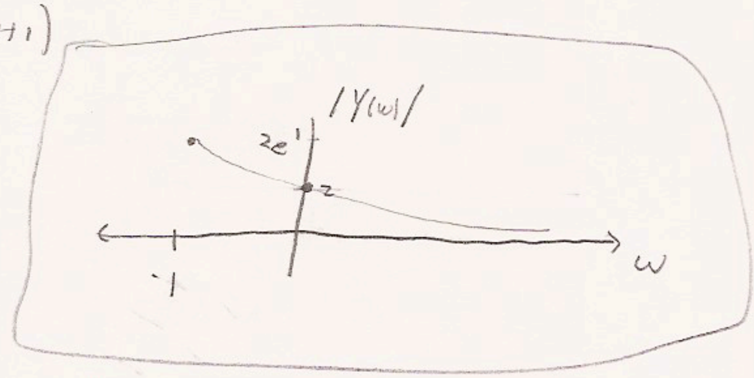
so $\mathcal{Y}(\omega) = \mathcal{F}(x(t-6))$

$\mathcal{Y}(\omega) = e^{-j\omega 6}\mathcal{X}(\omega)$, by (10) ✓

$= e^{-j6\omega}\left(-2e^{(j-1)\omega}u(\omega+1)\right)$

$= -2e^{-j6\omega+j\omega-\omega}u(\omega+1)$

$\boxed{= -2e^{-j5\omega-\omega}u(\omega+1)}$



$j5+1$

$= 2\left|e^{-j5\omega-\omega}\right|$ for $\omega \geq -1$, 0 else.

$= 2\left|e^{-j5\omega}\right|\left|e^{-\omega}\right|$ for $\omega \geq -1$

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