(22 pts) 1. Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No
If $y(t) = x(2t)$, is the system causal?		X
If $y(t) = (t+2)x(t)$, is the system causal?		X
If $y(t) = x(-t^2)$, is the system causal? present past	X	
If $y(t) = x(t) + t - 1$, is the system memoryless?	X	
If $y(t) = x(t^2)$, is the system memoryless?		X
If $y(t) = x(t/3)$, is the system stable?	X	
If $y(t) = tx(t/3)$, is the system stable?		X
If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable?	X	
If $y(t) = \sin(x(t))$, is the system time invariant?		X
If $y(t) = u(t) * x(t)$, is the system LTI?	X	
If $y(t) = (tu(t)) * x(t)$, is the system linear?	X	

outside x(t)

plug in

$\int t u(t) x$

(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\,u(t-\tau+2)\,d\tau$$

but $u(\tau) = 0$ when $\tau < 0$

$$= \int_{0}^{\infty} e^{-\tau}u(t-\tau+2)\,d\tau \;\; u(t)$$

but $u(t-\tau+2) = 0$

when $t-\tau+2 < 0$

$\iff -\tau < -t-2$

$\iff \tau > t+2$

$$= \int_{0}^{t+2} e^{-\tau}\,d\tau \;\; \cancel{u(t)} \quad -3$$

$u(t+2)$

$$= -e^{-\tau}\Big|_{0}^{t+2} u(t) = -e^{-t+2} + 1 \;\; u(t)$$

$\left(1 - e^{-(t+2)}\right)u(t)$

Please organize your answer so one can understand your computation easily. Like this it is very hard to follow. In particular, you must write "=" when you mean "equal".

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(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$. $= \frac{3}{1+j} e^{jt}$

$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

$E_\infty = \int_{-\infty}^{\infty} 1^2 \, dt = \infty$

$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$= \frac{1}{2T} \int_{-T}^{T} 1 \, dt = \frac{1}{\cancel{2T}} \cancel{2T} = 1$

$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$|x(t)| = \left| \frac{3e^{jt}}{1+j} \right|$

$= \frac{3 |e^{jt}|}{|1+j|} = \frac{3}{\sqrt{2}}$

$E_\infty = \infty$

$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} 9/2 \, dt$

$= \lim_{T \to \infty} \frac{1}{2T} \frac{9}{2} 2T = 9/2$

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(15 pts) **4.** Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$

(Simplify your answer as much as possible.)

$$T=4, \quad \sin(\pi t) = \frac{e^{j\pi} - e^{-j\pi}}{2j}$$

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\left(\frac{2\pi}{T}\right)t}\, dt$$

$$\frac{2\pi}{4} = \frac{\pi}{2}$$

$$x(t) = \frac{e^{j\pi/2}}{2j} - \frac{e^{-j\pi/2}}{2j}$$

$$\cancel{a_0 = \frac{1}{T}\int_{-\infty}^{\infty} |x(t)|\, dt}$$

$$a_4 = \frac{1}{2j} \qquad a_{-4} = \frac{-1}{2j}$$

5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

$$
\begin{array}{rcl}
\text{input} & & \text{output} \\
x_0[n] = \delta[n] & \rightarrow & y_0[n] = \delta[n-1], \\
x_1[n] = \delta[n-1] & \rightarrow & y_1[n] = 4\delta[n-2], \\
x_2[n] = \delta[n-2] & \rightarrow & y_2[n] = 9\delta[n-3], \\
x_3[n] = \delta[n-3] & \rightarrow & y_3[n] = 16\delta[n-4], \\
& \vdots & \\
x_k[n] = \delta[n-k] & \rightarrow & y_k[n] = (k+1)^2\delta[n-(k+1)] \text{ for any integer k.}
\end{array}
$$

(10 pts) a) Can this system be time-invariant? Explain.

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$x[n] \rightarrow \boxed{syst} \xrightarrow{y[n]=\delta[n-k]} \boxed{TD} \rightarrow z[n] = y[n-k]$

Not TI

$y[n] = \delta[n-k+n_0]$

$x[n] \rightarrow \boxed{TD} \xrightarrow{y[n]=\delta[n-n_0]} \boxed{syst} \rightarrow z[n] = y[n-n_0]$

$y[n] = \delta[n-k-n_0]$) $\neq$

(10 pts) b) Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?

$x[n] = \delta[n]$

$k=0$ $\quad y[n] = (k+1)^2 \delta[n-(k+1)]$

0

$y[n] = (0+1)^2 \delta[n-(0+1)]$

$y[n] = 1 \cdot \delta[n-1]$