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Time Shifting

We have $y(t) = x(t - t_0)$ $\leftarrow \in \mathbb{R}$

and $b_k = \frac{1}{T} \int_T x(t - t_0) e^{-jk\omega_0 t} dt$

if we let $\tau = t - t_0$

$\rightarrow \frac{1}{T} \int_T x(\tau) e^{-jk\omega_0(\tau + t_0)} d\tau$

$= \frac{e^{-jk\omega_0 t_0}}{T} \int_T x(\tau) e^{jk\omega_0 \tau} d\tau$

$= e^{-jk\omega_0 t_0} a_k = e^{-jk \frac{2\pi}{T} t_0} a_k$

$x(t - t_0) \leftrightarrow e^{-jk\omega_0 t_0} a_k = e^{-jk \left(\frac{2\pi}{T}\right) t_0} a_k$