
This is a 2 hour practice exam.

1. Let $f : X \rightarrow Y$ with X, Y both metric spaces. Prove that if for all $x, x_n \in X, x_n \rightarrow x$ we have $f(x_n) \rightarrow f(x)$ then f is continuous at x .

2. Let

$$f = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{else} \end{cases}$$

Find (with proof) the set on which f is continuous.

3. Proof or counter-example: the product of 2 uniformly continuous functions is uniformly continuous.

4. Find **without proof** a function who has a derivative (everywhere), but the derivative is discontinuous.

5. Converge absolutely? (with proof)

$$\sum_{n=100}^{\infty} \frac{\log^2(n^n) \sin(n^2 + 4n!)}{(n^2 - 4n + 7)^2}$$

6. For what x does the following sum converge?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{1 + x^n}$$

7. $f : \mathbb{R} \rightarrow \mathbb{R}$, continuous and

$$\lim_{|x| \rightarrow \infty} f(x) = 0 = \lim_{|x| \rightarrow \infty} f(1/x)$$

Show $f(\mathbb{R})$ is compact.

8. $f : \mathbb{R} \rightarrow \mathbb{R}, f(0) = 0$. For all $|x_n| \rightarrow \infty, \sum x_n f(1/x_n)$ converges. Show $f'(0)$ exists and is zero.