1.3 Exponential & sin function

Tuesday, August 28, 2007
3:03 PM

general form of exponential

$$x(t) = Ce^{at} \quad \text{where } C, a \text{ are complex}$$

$$x(t) = e^{st}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$ periodic w/ fund. period 2π

$$|e^{j\theta}| = 1$$

Not all # (complex) exponentials are periodic

$$x(t) = e^{(1+j)t}$$

<u>Trick</u>

$$\cos\omega t = Re(e^{j\omega t})$$

$$= \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\sin\omega t = Im(e^{j\omega t})$$

$$= \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

General Form for a DT Complex exponential

$$x[n] = Ca^n, \quad a, C \text{ are complex}$$

ex. $x[n] = j^{-n}$

$$x[n] = (1+3j)(1-4j)^n$$

or: $C = |C|e^{j\phi}$ frequency phase

$$a = |a| e^{j\omega}$$

$$x[n] = |c|\,|a|^n\, e^{j(\omega n + \phi)}$$

growth vs. decay $a < 1$ decay

Is $x[n] = e^{j\omega_0 n}$ periodic?

Recall $x(t) = e^{j\omega_0 t}$ is periodic.

Let's check:

we want $x[n+N] = x[n]$ for all n

$$e^{j\omega_0(n+N)} = \cancel{e^{j\omega_0 n}} \cdot e^{j\omega_0 N} = \cancel{e^{j\omega_0 n}}$$

$$e^{j\omega_0 N} = 1$$

$$\downarrow$$

$$\cos \omega_0 N + j \sin \omega_0 N = 1$$

if $\omega_0 N = k2\pi$ for some integer k

then $\cos = 1$ and $\sin = 0$

and it is periodic

$\frac{\omega_0}{2\pi} = \frac{k}{N}$ ← both integers, so $\frac{\omega_0}{2\pi}$ must be a rational number

ex. DT e^{jn} is not periodic because $\frac{\omega_0}{2\pi} = \frac{1}{2\pi}$ is not rational

$e^{j\pi n}$, $\frac{\omega_0}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2}$ is rational, so periodic

* basically ω_0 must be a multiple of π for signal to be periodic in DT. Also, the number multiplying π must be rational

We want to find the smallest $N \neq 0$ such that $x[n+N] = x[n]$ for all n

$$\Longleftrightarrow \omega_0 N = k2\pi \Longleftrightarrow N = \underline{k2\pi}$$

ω_0

look at $e^{\frac{1}{2}j\pi n}$ $\quad \omega_0 = \frac{1}{2}\pi$ $\quad N = \frac{k2\pi}{\frac{1}{2}\pi} = 4k$

$k=1$

smallest: $N=4$

Now look at $e^{3j\pi n}$ $\quad \omega_0 = 3\pi$

$N = \frac{k2\pi}{3\pi} = \frac{2}{3}k$ $\quad$ $k=3$ yields smallest period, $N=2$

The fund. Period of $x[n] = e^{j\omega_0 n}$ is $N_0 = \frac{k2\pi}{|\omega_0|}$

where k is the smallest possible integer that makes N an integer