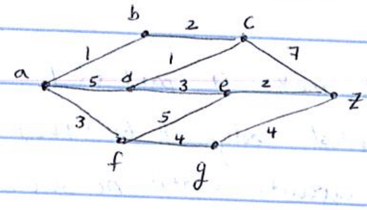
19 APRIL 2012

Dijkstra's Algorithm for Shortest Path.

e.g.



what is the shortest path $a \to z$?

$\boxed{\ }_i$

box = smallest

i = root?

a	$\boxed{0}$								
b	∞	$\boxed{1}_a$							
c	∞		$\boxed{1+2=3}_b$						
d	∞	0+5	take the smaller	4	$\boxed{4}_c$				
e	∞				8	$\boxed{7}_d$			
f	∞	0+3		$\boxed{3}_a$					
g	∞				7		$\boxed{7}_f$		
z	∞			10			9	11	$\boxed{9}_{\cancel{d}e}$

we slowly fill in the table to measure distance from a... of growing graph S.

* each colum's lowest # gets boxed (only 1 boxed # per column)

→ "Walking backward: $z \to e \to d \to c \to b \to a$

Algorithm

Input: G

Output: A shortest path.

Method: Initialize → a=0, all other verticies initialized to ∞.

S (subgraph of G) → {a} (we have not "discovered" other vert)

aim: completely explore G.

Iterate → (a) For each unexplored neighbor (unboxed) of any vertex in S, find the smallest value obtained as "value of the vertex (origin vert in Q) in X + distance to that vertex"

(b) of all numbers (on table of column), not attached to a vertex in S, find the smallest one. Add that vertex to S.

STOP: when z is in S.

shortest path alog: at most n^2 path step...

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Review Qs.

\# of subgraphs of a graph

$G =$  choices of vert 2^v.

$$1\begin{pmatrix} \cdot & \times \\ \times & \times \end{pmatrix} + 1\begin{pmatrix} \times & \times \\ \times & \cdot \end{pmatrix} + \begin{pmatrix} \cdot & \circ \\ \times & \times \end{pmatrix}\begin{pmatrix} \cdot & \times \\ \cdot & \times \end{pmatrix}\begin{pmatrix} \cdot & \times \\ \times & \cdot \end{pmatrix}\begin{pmatrix} \times & \cdot \\ \times & \cdot \end{pmatrix}\begin{pmatrix} \times & \cdot \\ \cdot & \times \end{pmatrix}\begin{pmatrix} \times & \cdot \\ \times & \cdot \end{pmatrix}$$

$$1 \quad 1 \quad 2 \quad 2 \quad 1 \quad 2 \quad 2 \quad 2 \quad + \ldots$$

thickness

$t(G) = \min k \{$ subgraph (planar) G_i with $G_i \cup G_2 \cup \ldots \cup G_k = G \}$.

then, $t \geq \frac{e}{3v-6}$.

Prf.

$3v - 6 \geq e$ if a graph is planar

We can arrange each edge of G to show in exactly one G_i (layer)

layers

G.

Let all G_i have all vert of G.

$\Rightarrow e = e_1 + e_2 + e_3 + \ldots + e_k$

$v = v_1 = v_2 = v_3 = \ldots = v_k$

Since G_i is planar

$3v_i - 6 \geq e_i$

so $3v_1 - 6 \geq e_1$

$3v_2 - 6 \geq e_2$

$\vdots$

$\underline{3v_k - 6 \geq e_k}$

$3(v_1 + v_2 + v_k) - 6k \geq e_1 + e_2 + \ldots e_k = e$

$3vk - 6k \geq e$

\# of layers $\longrightarrow k(3v - 6) \geq e$